MATEMATICA II
GUIA I
EJERCICIOS RESUELTOS
DE
CALCULO

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Maturín, Estado Monagas

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SIBIUDO

MATEMATICA II, GUIA I EJERCICIOS RESUELTOS DE CALCULO

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Lcdo. Rafael Figueroa

Lcdo. Marcos Ramírez

A manera de presentación

Cuando terminó el Semestre I-2011 en la Universidad, Rubén, Vinicio y Carmen decidieron no botar los mejores cuadernos de apuntes. Expresaron entonces la disposición de pasarlos a otros estudiantes para ayudarlos en los objetivos de sus estudios.

Así nació la idea de hacer unos cuadernos de ejercicios resueltos de Matemática para estudiantes de Ingeniería, orientados a facilitar la comprensión de los artificios y procedimientos que pueden encontrarse y seguirse cuando se resuelven problemas de Cálculo. En realidad, se omitieron algunas operaciones en el entendido que cada bachiller domina conocimientos precedentes de educación básica, sobre todo, los aprendizaje de operaciones con números reales, potenciación, radicación, operaciones con polinomios, productos notables, y factorización, para nombrar algunos aspectos.

Bajo esta motivación y con el entusiasmo de Mircia Dolores, no dudamos en ofrecerles la elemental colaboración para llevar a buen término su propósito.

Ramón R. Resplandor Moreno

Maturín, junio de 2015

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MATEMATICA II

FUNCIONES INVERSAS

$$Uso\ de\ la\ Derivada\ para\ probar$$
$$que\ la\ función\ inversa\ existe\ (soluciones\ pág. 14)$$

$1.-\ f(x)=4x-3$	$2.-\ f(x)=\frac{x}{3}+1$	$3.-\ f(x)=x^2+5$
$4.-\ f(x)=x^3+1$	$5.-\ f(x)=\ln(x-3)$	$6.-\ f(x)=\frac{x^4}{4}-2x^2$
$7.-\ f(x)=2-x-x^3$	$8.-\ f(x)=x^3-6x^2+9x$	$9.-\ f(x)=(x^2-1)^2$
$10.-f(x)=2x^2+8x+7$	$11.-f(x)=2-x-x^3$	$12.-\ f(x)=(5x-1)^2$
$13.-\ f(x)=\sqrt{5x-1}$	$14.-\ f(x)=\frac{2x-3}{x+2}$	$15.-\ f(x)=\frac{3x-1}{x+2}$
$16.-\ f(x)=\frac{3x+4}{2x+6}$	$17.-\ f(x)=\frac{x+2}{1-3x}$	$18.-f(x)=\ x^3+3x-1$
$19.-\ Si\ f(x)=6-\ x-x^3$		

$$Demostrar\ que\ \ f\left(f^{-1}(x)\right)=x\ \ y\ \ que\ \ f^{-1}\left(f(x)\right)=x\ \ (pág. 22)$$

$1.-f(x)=\sqrt{x}+1$	$2.-f(x)=1+\left(x^{\frac{1}{2}}+2\right)^{\frac{5}{3}}$	3.-$f(x)=\ 1+\left(x^{\frac{3}{2}}-2\right)^{\frac{5}{3}}$
$4.-f(x)=\sqrt{(x^3-2)^3+5}$	$5.-f(x)=\sqrt{1-(x-2)^3}$	$6.-f(x)=\left[\left(1+\frac{1}{x^3}\right)^5+1\right]^{\frac{3}{5}}$
$7.-\ f(x)=\left[\sqrt[3]{x}+1\right]^5$	$8.-\ f(x)=\left[\frac{2}{3}-\sqrt{3x+1}\right]^5$	$9.-\ f(x)=\frac{\sqrt[3]{x^{\frac{1}{3}}+2+8}}{\sqrt[3]{x^{\frac{1}{3}}+2+9}}$
$10.-\ f(x)=e^{\sqrt[3]{x+1}}$	$11.-\ f(x)=5^{\left(\sqrt{3x+1}\right)^3}$	

$Gráfica\ de\ funciones\ trigonométricas$
$y\ sus\ inversas\ \ (soluciones\ pág.35).$

$1.-\quad y=\text{sen}(x)$	$2.-\quad y=\cos(x)$	$3.-\quad y=\tan(x)$
$4.-\quad y=\sec\,(x)$	$5.-\quad y=\csc(x)$	$6.-\quad y=\cot\,(x)$

$Amplitud, período, desfase, dominio,$
$rango, inversa\ y\ gráfica\ de\ funciones$
$trigonométricas\ (soluciones\ pág.41).$

$1.-\ y=\text{sen}(x)$	$2.-\ y\ \cos(x)$	$3.-\ y=\cos\left(x+\frac{\pi}{2}\right)$
$4.-\ y=\text{sen}\left(\frac{\pi x}{4}-\pi\right)$	$5.-\ y=\tan\left(2x+\frac{\pi}{3}\right)$	$6.-\ y=\frac{3}{4}+\text{sen}\left(\frac{3x}{2}-\frac{\pi}{6}\right)$
$7.-\ y-3=\cos\left(2x-\frac{\pi}{3}\right)$	$8.-\ y=1+\text{sen}\left(\frac{\pi x}{3}\right)$	$9.-y=2+3\cos\left(\frac{\pi x}{2}-\pi\right)$
$10.-\ y=1+3\tan\left(\frac{x+\pi}{6}\right)$		

$Derivar\ y\ simplificar\ \ las$
$siguientes\ funciones\ (soluciones\ pág.59)$

$1.-\quad y=\text{sen}\left(3x+\frac{\pi}{2}\right)$	$2.-\quad y=(3x^2-1)^2$	$3.-\quad y=x^2e^x$
$4.-\quad y=4x^3-3\text{x}$	$5.-\quad y=\ln(x+e^x)$	$6.-y=\ln(x^2.e^x)$
$7.-y=\sqrt[3]{1-6x^5}$	$8.-y=\frac{a}{x}+\frac{x}{a}$	$9.-2x^2-3xy+4y^2=\sqrt{3}$
$10.-x^4=\frac{x-y}{x+y}$	$11.-\quad x^2y-xy=1-3x^2$	$12.-y=\sqrt{\text{sen}(x)}$
$13.-\ y=2\text{sen}(3x)\cos(3x)$	$14.-\ x^2\cos^3(4x)=x^3-y^3$	$15.-y=\text{sen}\left(\cos\sqrt{x^3}\right)$
$16.-\ y=\sec^2(x)\tan^2(x)$	$17.-\ y=\frac{x^2+1}{\sqrt{1-x^2}}$	$18.-\ y=\sec(xy)=xy$
$19.y=(2x^3-1)^2\sqrt[4]{x^2-3}$	$20.-\quad y^2\cos(x)=a^2\text{sen}(bx)$	

$$Derivar\ y\ simplificar\ las\ funciones\ que$$
$$se\ muestran\ a\ continuación\ (soluciones\ pág.\ 64)$$

$1.-y=\operatorname{sen}\left(7x-\frac{\pi}{2}\right)$	$2.-y=x^2e^{x^2}$	$3.-y=\sec^{-1}(3x^2+2)^5$
$4.-\ y=\log\left(\sqrt{x}-1\right)$	$5.-\ y=\operatorname{sen}^{-1}\left(e^{x^2}\right)$	$6.-\ y=\operatorname{sen}(\cos^{-1}(x+1))$
$7.-y=e^{\sqrt{x}}$	$8.-\ y=\ln(x+e^x)$	$9.-\ \tan^{-1}\left(\frac{x}{y}\right)+2x^2y=\frac{y}{x}$
$10.-y=\ln\left(\ln\left(\sqrt{x}\right)\right)$	$11.-\ \operatorname{sen}^{-1}(xy)=\cos^{-1}(x+y)$	$12.\operatorname{sen}^{-1}\left(\sqrt{x}\right)=\sec^{-1}\left(\sqrt{y}\right)$
$13.-$ $y=\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$	$14.-$ $y=\sqrt{x^2-1}+\sec^{-1}\left(\sqrt{x^2-1}\right)$	$15.-\ y=\frac{\ln^2(x)}{\ln(x)}$
$16.-y=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\operatorname{sen}^{-1}\left(\frac{x}{a}\right)$	$17.-\ y=\ln(x^2e^x)$	$18.-\ y=\ln(\tan^{-1}(e^{3x}))$
$19.-y=\ln\sqrt[5]{\frac{1+\operatorname{sen}x}{1-\operatorname{sen}x}}$	$20.-\ln(x+y)^3=x^2+y^2$	$21.-\ln(x+y)=e^{x.y}+x^2$
$22.-y=\frac{e^x-e^{-x}}{e^x+e^{-x}}$	$23.-\ln(x^2+y^2)=2\tan^{-1}\left(\frac{x}{y}\right)$	$24.-\ln(y)=\operatorname{sen}(x)$
$25.-$ $y=\operatorname{sen}^{-1}(\ln x)-\ln(\tan^{-1}x)$	$26.-\sqrt{1+\ln y}=\operatorname{sen}^2(3x-1)$	$27.-\ln[(2x+1)^3(x^2-4)^3]$
$28.\ y=\operatorname{sen}^3(e^x)\cos^4(e^{-x})$	$29.-\ln\sqrt{xy}+\ln\sqrt{x+y}=4$	$30.-\operatorname{sen}^{-1}[\ln(x+y)]=x.y$
$31.-y=10e^{-x}\cos(x)$		

$$Derivar\ y\ simplificar\ las\ siguientes$$
$$funciones\ hiperbólicas\ (soluciones\ p.\ 75)$$

$1.-y=x\ \operatorname{senh}x+e^{\cosh x}$	$2.-y=\operatorname{senh}(\tan^{-1}(e^{3x}))$	$3.-y=\frac{\operatorname{senh}x}{1+\cosh x}$
$4.-y=\operatorname{sen}^{-1}(\tanh(x^2))$	$5.-\operatorname{senh}y=\tan x$	$6.-\ y=\ \operatorname{arccoth}\left[\ln\left(\frac{x}{2}\right)\right]$
$7.-y=\ln\left(\sqrt{\tanh(2x)}\right)$	$8.-y=\left[\sqrt[3]{\operatorname{sech}x}\right]^4$	$9.-$ $y=\ln[\coth(5x)-\operatorname{csch}(5x)]$
$10.-$ $\tanh^{-1}y=\tan^{-1}(\operatorname{sech}x)$	$11.-y=\ln^3\left(\tanh\left(e^{x^2}\right)\right)$	$12.-y=\cosh^{-1}\left(e^{x^2}\right)$
$13.-y=\ln(\ln(\operatorname{sech}^{-1}(x^2)))$	$14.-y=\ln[\operatorname{senh}^{-1}(e^{2x})]^{3/2}$	$15.-$ $y=\sqrt[5]{\operatorname{csch}^2(x^2-3x+1)}$

16. $- \\ y=[\operatorname{sech}^3(3x).\cosh^3(3x)]$	17. $-y=\frac{1}{x}\operatorname{sech}\left(\frac{1}{x}\right)$	8. $-\operatorname{senh}^{-1}(x^2+y^2)=\tanh^{-1}x$
19. $-\ y=\ln\left(\operatorname{senh}\left(\sqrt{x}\right)\right)$	20. $-\ y=\frac{1}{\sqrt{\tanh\left(\ln^2(3x)\right)}}$	

$$Calcular\ las\ siguientes\ integrales\ (soluciones\ pág. 81)$$

1. $-\int(\sqrt{x}+8x-3)dx$	2. $-\int(2\sqrt{x}-\sqrt[3]{x}-x^4)dx$	3. $-\int\frac{2dx}{x^3}$
4. $-\int\frac{dx}{\sec(x)}$	5. $-\int\left[\sqrt{x}\,(x+\sqrt[5]{x})\right]dx$	6. $-\int\frac{(x^2+2x)^2dx}{x^2}$
7. $-\int x\sqrt{x^2+2}\,dx$	8. $-\int\tan(x)dx$	9. $-\int\frac{(x^7+5x^5-6x^3+8)dx}{x^2}$
10. $-\int(4x+3)^2\,dx$	11. $-\int\frac{x^2dx}{x^3-2}$	12. $-\int 2xe^{x^2}dx$
13. $-\int\frac{dx}{1-x}$	14. $-\int\frac{(x+2)dx}{2\sqrt{x+2}}$	15. $-\int(3x^2-1)^3dx$
16. $-\int\left(\frac{5}{x}-\frac{x}{5}\right)dx$	17. $-\int\cos(x^2)2x\,dx$	18. $-\int(1-\cos\,x)^3\operatorname{sen}x\,dx$
19. $-\int\frac{x^2dx}{x^2-2}$	20. $-\int(3x^2-1)(x+2)dx$	21. $-\int\frac{(5x+3)^2dx}{4}$
22. $-\int(5x^3-1)^2dx$	23. $-\int e^{5x+3}\,dx$	24. $-\int(2x^2+1)^{1/2}x^3dx$
25. $-\int\frac{(x+3)dx}{(3-x)^{2/3}}$	26. $\int(1+\operatorname{sen}2x)^{3/2}\cos 2x\,dx$	27. $-\int\frac{\operatorname{sen}x\,dx}{\cos^2x}$
28.- $\int(x^{-2}+x)\,(1-2x^{-3})dx$	29. $-\int\frac{xdx}{(1-2x^2)^3}$	30. $-\int\frac{(x^2+x)dx}{(4-3x^2-2x^3)}$
31. $\int\sqrt{x^4+x^2}\,(10x^3+5x)\,dx$	32. $-\int\frac{dx}{\sqrt{x}(\sqrt{x}-3)^5}$	33. $-\int(1+\frac{1}{x})\frac{1}{x^2}dx$
34. $-\int\frac{(\sqrt{x}-1)^{\frac{3}{2}}dx)}{\sqrt{x}}$	35. $-\int\frac{\left(x^{1/3}-2\right)^5}{\sqrt[3]{x^2}}dx$	36. $-\int x^2\,(x+1)^{1/2}\,dx$
37. $-\int\frac{3xdx}{(1-3x)^2}$	38. $-\int(3-x)^{5/2}\,xdx$	39. $-\int\sqrt{3+x}\,(x+1)^2dx$
40. $- \\ \int x^{-3}\,(1+\frac{1}{2x})^{3/2}\,dx$	41. $- \\ \int\frac{(x+1)dx}{\sqrt{x^2+2x+2}}$	42. $- \\ \int\frac{(x+3)dx}{\sqrt[3]{(9-x)^2}}$
43. $- \\ \int\left(x+\frac{1}{x}\right)^{5/3}\left(\frac{x^2-1}{x^2}\right)dx$	44. $-\int(2x^2-1)^{1/3}x^3dx$	45. $-\int\frac{1}{x^2}\,(\frac{x-1}{x})^{1/2}\,dx$

46.- $\int \frac{\sqrt{2x^2+3}}{x^{-3}}dx$	47.- $\int x(x^2+2)^{10}dx$	48.- $\int \frac{(2x+3)dx}{\sqrt{x^2+3x}}$
49.- $\int \sqrt{x}\sqrt{1+x\sqrt{x}}\,dx$	50.- $\int x^3\,(x^2-3)^{1/2}\,dx$	51.- $\int \frac{xdx}{(ax+b)^{3/2}}$
52.- $\int \left(\frac{x^3+2}{x^2}\right)\sqrt{x-\frac{1}{x^2}}\,dx$	53.- $\int \frac{dx}{\sec(2x)}$	54.- $\int \frac{\cot(3x)dx}{\mathrm{sen}^2(3x)}$
55.- $\int x^2\cot(x^3)\csc(x^3)\,dx$	56.- $\int \sqrt{\tan(x)}\sec^2(x)dx$	57.- $\int \sec x(\sec x-\tan x)dx$
58.- $\int (\mathrm{sen}(x))^{2/3}\cos(x)\,dx$	59.- $\int \frac{\tan x\,dx}{\cos^2 x}$	60.- $\int \frac{\mathrm{sen}(2x)dx}{\sqrt{1+\mathrm{sen}^2 x}}$
61.- $\int \frac{dx}{\cos^2 x\,\sqrt{\tan(x)-1}}$	62.- $\int \frac{\csc^2(\sqrt{x})dx}{\sqrt{x}}$	63.- $\int \frac{x^2\mathrm{sen}(x^3+1)dx}{\cos^3(x^3+1)}$
64.- $\int \frac{dx}{\mathrm{sen}^2(ax+b)}$	65.- $\int x\,\mathrm{sen}\left(\frac{x^2}{2}\right)dx$	66.- $\int \frac{\mathrm{sen}\sqrt{x}.\cos\sqrt{x}\,dx}{\sqrt{x}}$
67.- $\int \sec^3 x\tan x\,dx$		

Integrales que conducen a funciones trigonométricas inversas (soluciones pág. 98)

1.- $\int \frac{dx}{\sqrt{2-5x^2}}$	2.- $\int \frac{dx}{\sqrt{2+x-x^2}}$	3.- $\int \frac{dx}{x^2+2x+10}$
4.- $\int \frac{xdx}{x^4+3}$	5.- $\int \frac{dx}{x\sqrt{4x^2-9}}$	6.- $\int \frac{x^2dx}{\sqrt{1-x^6}}$
7.- $\int \frac{xdx}{x^2\sqrt{x^4-1}}$	8.- $\int \frac{ax\,dx}{x^4+b^4}$	9.- $\int \frac{(x+2)dx}{\sqrt{4-2x-x^2}}$
10.- $\int \frac{dx}{x^2+10x+30}$	11.- $\int \frac{dx}{\sqrt{4-(x+2)^2}}$	12.- $\int \frac{\sec(x)\tan(x)dx}{9-4\sec^2(x)}$
13.- $\int \frac{\cos(x)dx}{1+\mathrm{sen}^2(x)}$	14.- $\int \frac{dx}{\sqrt{20+8x-x^2}}$	15.- $\int \frac{dx}{\sqrt{2x-x^2}}$
16.- $\int \frac{dx}{\sqrt{x}(1+x)}$	17.- $\int \frac{dx}{x\sqrt{1-\ln^2(x)}}$	18.- $\int \frac{e^x dx}{\sqrt{1-e^{2x}}}$
19.- $\int \frac{x^2dx}{x^6+25}$	20.- $\int \frac{\cos(x)\mathrm{sen}(x)dx}{1+\mathrm{sen}^4(x)}$	21.- $\int \frac{dx}{4x^2-4x+2}$
22.- $\int \frac{2\,\mathrm{sen}(4x)dx}{1+\mathrm{sen}^4(2x)}$	23.- $\int \frac{\csc^2(3x)dx}{25+\cot^2(3x)}$	24.- $\int \frac{dx}{\sqrt{12-4x-x^2}}$
25.- $\int \frac{dx}{\sqrt{1-a^2x^2}}$	26.- $\int \frac{dx}{\sqrt{9-4x^2}}$	

Integralesqueconducenaexpresiones logarítmicas o exponenciales (soluciones pág. 106).

$1.-\int \frac{xdx}{1+x^2}$	$2.-\int xe^{x^2}dx$	$3.-\int \frac{dx}{x\ln x}$
$4.-\int \frac{\ln(x)dx}{x}$	$5.-\int \frac{\cos(\ln(x))dx}{x}$	$6.-\int \frac{\text{sen}(x)dx}{\cos(x)}$
$7.-\int \tan(x)dx$	$8.-\int \frac{(e^x+1)dx}{e^x}$	$9.-\int \frac{e^x dx}{(e^x-5)}$
$10.-\int \sec^2(x)e^{\tan(x)}dx$	$11.-\int \frac{(e^x+\sec^2 x)dx}{(\tan x+e^x)}$	$12.-\int \frac{(e^{2x}+e^x+1)dx}{e^x}$
$13.-\int \frac{\sqrt{x}dx}{1+x\sqrt{x}}$	$14.-\int \frac{e^x dx}{4-e^x}$	$15.-\int \frac{e^{\sqrt{x+1}}dx}{\sqrt{x+1}}$
$16.-\int \frac{(e^x+e^{-x})dx}{(e^x-e^{-x})}$	$17.-\int \frac{dx}{x\ln\sqrt{x}}$	$18.-\int \frac{dx}{e^x+1}$
$19.-\int \frac{\left(e^{\sqrt{x}}+1\right)dx}{\sqrt{x}}$	$20.-\int \frac{\ln(x)dx}{x(1+\ln(x))^{1/2}}$	$21.-\int e^{3x}e^{2x}dx$
$22.-\int \frac{e^{-x}dx}{e^{-x}+1}$	$23.-\int (2x+1)10^{(3x^2+3x+1)}\,dx$	$24.-\int (e^{-3x}+\frac{1}{e^x})dx$
$25.-\int \frac{e^{3x}dx}{(1-2e^{3x})^2}$	$26.-\int a^{x\ln x}(1+\ln x)dx$	$27.-\int \frac{e^{2x}dx}{e^{4x}+3}$
$28.-\int \frac{(2+\ln^2 x)dx}{x(1-\ln x)}$	$29.-\int (e^{\frac{x}{2}}-e^{\frac{-x}{3}})dx$	$30.-\int (x-1)e^{-x^2+2x}dx$
$31.-\int \frac{dx}{e^x+e^{-x}}$	$32.-\int (e^{x/2}-e^{-x/2})dx$	$33.-\int \frac{dx}{\sqrt{e^x}}$
$34.-\int \frac{dx}{3-x}$	$35.-\int \frac{dx}{x(\ln x)^4}$	$36.-\int \frac{e^{2x}dx}{\sqrt{e^{4x}+1}}$
$37.-\int \frac{e^{x^{-2}}}{x^3}dx$	$38.-\int e^{-x^2+2}\,xdx$	$39.-\int x^2e^{x^3}dx$
$40.-\int e^{\text{sen}^2(x)}\,\text{sen}(2x)dx$	$41.-\int \frac{\sec^5 x\,dx}{\csc x}$	$42.-\int \frac{\cot(\ln x)dx}{x}$

Ecuaciones diferenciales elementales (soluciones pág. 117)

$1.-\frac{dy}{dx}=6-3x^2$	$2.-\frac{dy}{dx}=3x^2y^2$	$3.-\frac{dy}{dx}=x^{-2}-x$
$4.-\frac{dy}{dx}=\frac{x^2\sqrt{x^3-3}}{y^2}$	$5.-\frac{dy}{dt}=(2t+1)^2$	$6.-\frac{dy}{dt}=3t^3+4t-6$
$7.-\frac{dy}{dx}=\frac{(x+2)^2}{(3-y)}$	$8.-\quad y''=\sqrt{2x-3}$	$9.-\frac{d^2y}{dx^2}=5x^2+1$

10. $-x^3\dfrac{dy}{dx} = y^2(x-4)$	11. $-\dfrac{d^2y}{dx^2} = -9\,\mathrm{sen}(3x)$	12. $-\ x^2y\dfrac{dy}{dx} = (y^2-1)^{\frac{3}{2}}$
13. $-\ \dfrac{dy}{dx} = (x+1)(x+2)$	14. $-$ $(y+5)dx-(x-2)dy=0$	15. $-$ $xydx-(1+x^2)dy=0$
16. $-(x-1)dy+\ y^2dx=0$	17. $-$ $(x^2-2)dy+(2y+3)xdx = 0$	18. $-$ $(x^2+4)xydy(y^2+3)dx=0$
19. $-$ $\cos^2 y\,dx+\mathrm{sen}\,x\,dy=0$	20. $-$ $(3-y)dx+2xydy=0$	

FUNCIONES INVERSAS

Definición:

Si f *es una función uno a uno formada por los pares* (x, y), *entonces existe un función,*

f^{-1}, *llamada función inversa de* f, *tal que* f^{-1} *es el conjunto de pares* (y, x)

definidos por $x = f^{-1}(y) \quad \Leftrightarrow \quad y = f(x)$.

$$El\ dominio\ de\ f^{-1}\ \ es\ \ el\ \ rango\ de\ \ f$$

$$El\ rango\ de\ f^{-1}\ \ es\ \ el\ dominio\ de\ \ f$$

1.– *Según la definición se tiene que* $f^{-1}(y) = x$, *pero como* $y = f(x)$ *se tendrá*

$$f^{-1}[f(x)] = x$$

2.– *Nuevamente, por definición* $f(x) = y$, *pero* $x = f^{-1}(y)$ *luego*

$$f[f^{-1}(y)] = y \quad es\ decir \quad f[f^{-1}(x)] = x$$

Sea $f(x)$ *una función que tiene como dominio el intervalo cerrado* $[a, b]$, *entonces*:

1.– *Si* $f(x)$ *es continua y creciente en* $[a, b]$, $f(x)$ *tiene una inversa* $f^{-1}(x)$ *que está definida*

$$en\ [f(a), f(b)]$$

2.– *Si* $f(x)$ *es continua y decreciente en* $[a, b]$, f *tiene una inversa* $f^{-1}(x)$ *que está definida*

$$en\ [f(a), f(b)]$$

Una función que siempre es creciente o decreciente en un intervalo, se denomina función monótona en ese intervalo. Las funciones monótonas son uno a uno.

Para las funciones continuas en $[a, b]$ *se cumple que*:

$a)$ f *es creciente si* $f'(x) > 0$ *en ese intervalo.*

$b)$ f *es decreciente si* $f'(x) < 0$ *en ese intervalo.*

Hay funciones que tienen inversa, pero no podemos obtener una ecuación que defina explícitamente la función inversa. Por ejemplo, si $f(x) = x^5 + x^3 + 10x$, *entonces* $f'(x) = 5x^4 + 3x^2 + 10$. *Como* $f'(x) > 0$ *para todo valor de* x, f *es una función continua y creciente y por ello tiene inversa.*

Sea f *continua y monótona en* $[a,b]$. *Considérese* $c \in [a,b]$ *y* $f(c) = d$.

Si $f'(c)$ *existe y* $f'(c) \neq 0$, *entonces* $(f^{-1})'(d)$ *existe y* $(f^{-1})'(d) = \dfrac{1}{f'(c)}$.

NOTA: La gráfica de f *y de su inversa* f^{-1} *son simétricas respecto a la bisectriz del primero y tercer cuadrantes del sistema de coordenadas.*

Uso de la Derivada para probar que la función inversa existe.

EJERCICIOS

1. —*Usar la derivada para determinar si la función* $f(x) = 4x - 3$ *tiene inversa.*

Solución: dom $f = R$ *y* $f(x) = 4x - 3 \;\Rightarrow\; f'(x) = 4$

así que $f'(x) > 0$, $\forall x \in Dom\, f$, *luego* f *es continua y creciente en* R, *por lo tanto tiene inversa en su dominio.*

2. —*Usar la derivada para determinar si la función* $f(x) = \dfrac{x}{3} + 1$ *tiene inversa.*

Solución: dom $f = R$ *y* $f(x) = \dfrac{x}{3} + 1 \;\Rightarrow\; f'(x) = \dfrac{1}{3}$

así que $f'(x) > 0$, $\forall x \in Dom\, f$, *luego* f *es continua y creciente en* R, *por lo tanto tiene inversa en su dominio.*

$3. -Usar\ la\ derivada\ para\ determinar\ si\ la\ función\ f(x) = x^2 + 5\ \ tiene\ inversa.$

$Solución:\ dom\ f = R\ \ y\ \ \ f(x) = x^2 + 5\ \ \ \Rightarrow\ \ \ f'(x) = 2x$

$así\ que\ \ f'(x) > 0,\ \ \forall x \epsilon (0, \infty)\ \ \ y\ \ \ f'(x) < 0,\ \ \forall x \epsilon (-\infty, 0)$

$luego\ f\ es\ continua\ \ pero\ no\ \ es\ monótona\ en\ su\ dominio.\ \ Por\ lo\ tanto\ \ \ no\ \ \ tiene\ inversa$

$en\ su\ dominio$

$4. -Usar\ la\ derivada\ para\ determinar\ si\ la\ función\ f(x) = x^3 + 1\ \ tiene\ inversa.$

$Solución:\ dom\ f = R\ \ \ y\ \ \ f(x) = x^3 + 1\ \ \ \Rightarrow\ \ \ f'(x) = 3x^2$

$así\ que\ \ f'(x) > 0,\ \ \forall x \epsilon (-\infty, \infty),\ \ luego\ f\ es\ continua\ y\ \ monótona\ en\ su\ dominio.$

$Por\ lo\ \ tanto\ f\ \ tiene\ inversa\ en\ su\ dominio$

$5. -Usar\ la\ derivada\ para\ determinar\ si\ la\ función\ f(x) = \ln(x-3)\ \ tiene\ inversa.$

$Solución:\ dom\ f = (3, \infty)\ \ \ y\ \ \ f(x) = \ln(x-3)\ \ \ \Rightarrow\ \ \ f'(x) = \dfrac{1}{x-3}$

$así\ que\ \ f'(x) > 0,\ \ \forall x \epsilon (3, \infty),\ \ luego\ f\ es\ continua\ y\ \ monótona\ en\ su\ dominio.$

$Por\ lo\ \ tanto\ f\ \ tiene\ inversa\ en\ su\ dominio$

$6. -Usar\ la\ derivada\ para\ determinar\ si\ la\ función\ f(x) = \dfrac{x^4}{4} - 2x^2\ \ tiene\ inversa.$

$Solución:\ dom\ f = R\ \ \ y\ \ \ f(x) = \dfrac{x^4}{4} - 2x^2\ \ \ \Rightarrow\ \ \ f'(x) = x^3 - 4x$

$$f'(x) = x(x^2 - 4) = x(x-2)(x+2)$$

$así\ que\ \ f'(x) > 0,\ \ \forall x \epsilon (-2,0) \cup (2, \infty)\ \ y$

$$f'(x) < 0,\ \ \forall x \epsilon (-\infty, -2) \cup (0,2)$$

$luego\ f\ es\ continua\ pero\ no\ es\ \ monótona\ en\ su\ dominio.\ \ Por\ lo\ \ tanto\ f\ no\ tiene\ inversa$

$en\ su\ dominio$

$7. -Usar\ la\ derivada\ para\ determinar\ si\ la\ función\ f(x) = 2 - x - x^3\ \ tiene\ inversa.$

$Solución:\ dom\ f = R\ \ y\ \ \ f(x) = 2 - x - x^3\ \ \Rightarrow\ \ \ f'(x) = -1 - 3x^2$

$así\ que\ \ f'(x) < 0,\ \ \forall x \in R$

$luego\ f\ es\ continua\ \ y\ \ monótona\ en\ su\ dominio.\ Por\ lo\ \ tanto\ f\ \ tiene\ inversa$

$en\ su\ dominio$

$8. -Usar\ la\ derivada\ para\ determinar\ si\ la\ función\ f(x) = x^3 - 6x^2 + 9x\ \ tiene\ inversa.$

$Solución:\ dom\ f = R\ \ y\ \ \ f(x) = x^3 - 6x^2 + 9x\ \ \Rightarrow\ \ \ f'(x) = 3x^2 - 12x + 9$

$$f'(x) = 3(x^2 - 4x + 3) = 3(x - 3)(x - 1)$$

$$así\ que\ \ f'(x) > 0,\ \ \forall x \in (-\infty, 1) \cup (3, \infty)\ \ y\ \ f'(x) < 0,\ \ \forall x \in\ (1,3)$$

$luego\ f\ es\ continua\ \ pero\ no\ es\ \ monótona\ en\ su\ dominio.\ Por\ lo\ \ tanto\ f\ no\ \ tiene\ inversa$

$en\ su\ dominio$

$9. -Usar\ la\ derivada\ para\ determinar\ si\ la\ función\ f(x) = (x^2 - 1)^2\ \ tiene\ inversa.$

$Solución:\ dom\ f = R\ \ y\ \ \ f(x) = (x^2 - 1)^2\ \ \Rightarrow\ \ \ f'(x) = 2(x^2 - 1)(2x)$

$$f'(x) = 4x(x - 1)(x + 1)$$

$$así\ que\ \ f'(x) < 0,\ \ \forall x \in (-\infty, -1) \cup (0,1)\ \ y\ \ f'(x) > 0,\ \ \forall x \in\ (-1,0) \cup (1, \infty)$$

$luego\ f\ es\ continua\ \ pero\ no\ es\ \ monótona\ en\ su\ dominio.\ Por\ lo\ \ tanto\ f\ no\ \ tiene\ inversa$

$en\ su\ dominio$

$10. -Dada\ f(x) = 2x^2 + 8x + 7\ \ \ y\ \ \ conociendo\ que\ x < -2,\ \ calcular\ \ (f^{-1})'(1)\ .$

$Solución:\ Si\ \ (c, d) \in f(x),\ \ entonces\ \ (d, c) \in f^{-1}(x)$

$$es\ decir,\ \ (1, c) \in f^{-1}(x)\ \Rightarrow\ \ (c, 1) \in f(x)$$

$$así\ que\ \ \ 2x^2 + 8x + 7 = 1\ \ \ \Rightarrow\ \ 2x^2 + 8x + 6 = 0$$

$$2(x^2 + 4x + 3) = 0\ \ \Rightarrow\ \ 2(x + 3)(x + 1) = 0\ \ \ \Rightarrow\ \ \ x = -1,\ \ x = -3$$

$$conociendo\ que\ debe\ ser\ \ \ x < -2,\ \ la\ solución\ será\ \ x = -3$$

$$f'(x) = 4x + 8 \;\Rightarrow\; f'(-3) = -12 + 8 = -4$$

$$(f^{-1})'(1) = \frac{1}{f'(-3)} = -\frac{1}{4}$$

$11.-Dada\ f(x) = 2 - x - x^3,\ \ calcular\ (f^{-1})'(2)\,.$

$Solución:\ Si\ \ (c,d) \in f(x),\ \ entonces\ \ (d,c) \in f^{-1}(x)$

$$es\ decir,\ \ (2,c) \in f^{-1}(x) \;\Rightarrow\; (c,2) \in f(x),\ \ así\ que\ 2 - x - x^3 \ = 2 \;\Rightarrow\; -x - x^3 = 0$$

$$-x(1 + x^2) = 0 \;\Rightarrow\; x = 0,\quad f'(x) = -1 - 3x^2 \;\Rightarrow\; f'(0) = -1$$

$$(f^{-1})'(2) = \frac{1}{f'(0)} = \frac{1}{-1} = -1$$

$12.-Dada\ f(x) = (5x-1)^2\ \ y\ \ conociendo\ que\ x \geq 0,\ \ calcular\ (f^{-1})'(1).$

$Solución:\ Si\ \ (c,d) \in f(x),\ \ entonces\ \ (d,c) \in f^{-1}(x)$

$$es\ decir,\ \ (1,c) \in f^{-1}(x) \;\Rightarrow\; (c,1) \in f(x)$$

$$así\ que\ \ (5x-1)^2 \ = 1 \;\Rightarrow\; 5x - 1 = 1 \;\Rightarrow\; x = \frac{2}{5}$$

$$como\quad x \geq 0,\ \ la\ solución\ es\ \ x = \frac{2}{5}\quad y\quad f'(x) = 10(5x-1) \Rightarrow\; f'\left(\frac{2}{5}\right) = 10$$

$$(f^{-1})'(1) = \frac{1}{f'\left(\frac{2}{5}\right)} = \frac{1}{10}$$

$13.-Dada\ f(x) = \sqrt{5x-1}\ ,\ \ calcular\ (f^{-1})'(1).$

$Solución:\ Si\ \ (c,d) \in f(x),\ \ entonces\ \ (d,c) \in f^{-1}(x)$

$$es\ decir,\ \ (1,c) \in f^{-1}(x) \;\Rightarrow\; (c,1) \in f(x)$$

$$así\ que\ \ \sqrt{5x-1} \ = 1 \;\Rightarrow\; 5x - 1 = 1 \;\Rightarrow\; x = \frac{2}{5}$$

$$f'(x) = \frac{5}{2\sqrt{5x-1}} \Rightarrow\; f'\left(\frac{2}{5}\right) = \frac{5}{2\sqrt{2-1}} = \frac{5}{2}$$

$$(f^{-1})'(1) = \frac{1}{f'\left(\frac{2}{5}\right)} = \frac{1}{\frac{5}{2}} = \frac{2}{5}$$

$14.-Dada\ f(x) = \dfrac{2x-3}{x+2}$, $calcular\ (f^{-1})'(0).$

$Solución$: $Si\ \ (c,d) \in f(x),\ \ entonces\ \ (d,c) \in f^{-1}(x)$

$$es\ decir,\ \ (0,c) \in f^{-1}(x) \ \Rightarrow\ \ (c,0) \in f(x)$$

$$así\ que\ \ \frac{2x-3}{x+2} = 0 \ \ \Rightarrow\ \ 2x-3=0 \ \ \Rightarrow\ \ x = \frac{3}{2}$$

$$f'(x) = \frac{7}{(x+2)^2} \Rightarrow\ f'\left(\frac{3}{2}\right) = \frac{7}{\left(\frac{3}{2}+2\right)^2} = \frac{4}{7}$$

$$(f^{-1})'(0) = \frac{1}{f'\left(\frac{3}{2}\right)} = \frac{1}{\frac{4}{7}} = \frac{7}{4}$$

$15.-Dada\ f(x) = \dfrac{3x-1}{x+2}$, $calcular\ (f^{-1})'(0).$

$Solución$: $Si\ \ (c,d) \in f(x),\ \ entonces\ \ (d,c) \in f^{-1}(x)$

$$es\ decir,\ \ (0,c) \in f^{-1}(x) \ \Rightarrow\ \ (c,0) \in f(x)$$

$$así\ que\ \ \frac{3x-1}{x+2} = 0 \ \ \Rightarrow\ \ 3x-1=0 \ \ \Rightarrow\ \ x = \frac{1}{3}$$

$$f'(x) = \frac{7}{(x+2)^2} \Rightarrow\ f'\left(\frac{1}{3}\right) = \frac{7}{\left(\frac{1}{3}+2\right)^2} = \frac{9}{7}$$

$$(f^{-1})'(0) = \frac{1}{f'\left(\frac{1}{3}\right)} = \frac{1}{\frac{9}{7}} = \frac{7}{9}$$

$16.-Dada\ f(x) = \dfrac{3x+4}{2x+6}$, $calcular\ (f^{-1})'(1).$

$Solución$: $Si\ \ (c,d) \in f(x),\ \ entonces\ \ (d,c) \in f^{-1}(x)$

$$es\ decir,\ \ (1,c) \in f^{-1}(x) \ \Rightarrow\ \ (c,1) \in f(x)$$

$$así\ que\ \ \frac{3x+4}{2x+6} = 1 \ \ \Rightarrow\ \ 3x+4 = 2x+6 \ \ \Rightarrow\ \ x = 2$$

$$f'(x) = \frac{10}{(2x+6)^2} \Rightarrow \; f'(2) = \frac{10}{(4+6)^2} = \frac{1}{10}$$

$$(f^{-1})'(1) = \frac{1}{f'(2)} = \frac{1}{\frac{1}{10}} = 10$$

$17.-Dada\ f(x) = \dfrac{x+2}{1-3x}$, $calcular\ (f^{-1})'(1).$

$Solución$: $Si\ \ (c,d) \in f(x),\ \ entonces\ \ (d,c) \in f^{-1}(x)$

$$es\ decir,\ \ (1,c) \in f^{-1}(x)\ \Rightarrow\ (c,1) \in f(x)$$

$$así\ que\ \ \frac{x+2}{1-3x} = 1 \quad \Rightarrow \quad x+2 = 1-3x \quad \Rightarrow \quad x = -\frac{1}{4}$$

$$f'(x) = \frac{7}{(1-3x)^2} \Rightarrow \; f'\left(-\frac{1}{4}\right) = \frac{7}{\left(1+\frac{3}{4}\right)^2} = \frac{7}{\frac{49}{16}} = \frac{16}{7}$$

$$(f^{-1})'(1) = \frac{1}{f'\left(-\frac{1}{4}\right)} = \frac{1}{\frac{16}{7}} = \frac{7}{16}$$

$18.-Si\ f(x) = \ x^3 + 3x - 1$:

$I.-Demuestre\ que\ f\ tiene\ inversa$

$II.-Calcule\ la\ pendiente\ de\ la\ recta\ tangente\ a\ la\ gráfica\ de\ f\ en\ el\ punto$ $(1,3)$.

$III.-Calcule\ la\ pendiene\ de\ la\ recta\ tangente\ a\ la\ gráfica\ de\ f^{-1}(x) en\ el\ punto$ $(3,1)$.

$IV.-Trazar\ las\ gráficas\ de\ \ f\ \ y\ \ f^{-1}\ \ en\ el\ mismo\ sistema\ de\ coordenadas.$

$Solución\ I$: $Dom\ F = R,\qquad f(x) = \ x^3 + 3x - 1 \quad \Rightarrow \quad f'(x) = 3x^2 + 3$

$f'(x) > 0\ \ \forall x \in Dom\ f,\ \ luego\ \ f\ es\ continua\ y\ monótona\ en\ Dom\ f\ \ y\ \ por\ lo\ \ tanto$ $tiene\ inversa.$

$Solución\ II$: $f(x) = \ x^3 + 3x - 1 \quad \Rightarrow \quad f'(x) = 3x^2 + 3,\ \ así\ \ que\ la\ pendiente\ de\ la$ $recta\ tangente\ a\ la\ gráfica\ de\ f\ en$ $(1,3)$ $es\ \ f'(1) = 6.$

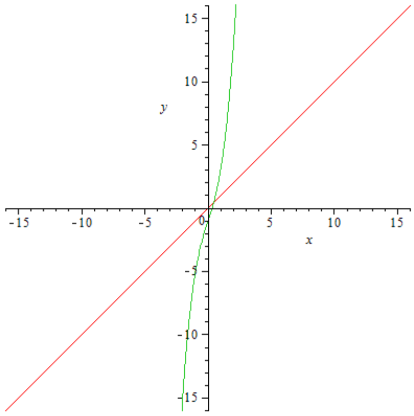
$Solución\ III$: $Pendiente\ de\ la\ recta\ tangente\ a\ la\ gráfica\ de\ \ f^{-1}(x)\ \ en\ \ \ (3,1)\ \ es$

$$[f^{-1}(3)]' = \frac{1}{f'(1)} = \frac{1}{6}$$

$Solución\ IV$: $Para\ trazar\ las\ gráficas\ de\ f\ \ y\ \ f^{-1}\ en\ el\ mismo\ \ sistema\ de\ coordenadas$ $se\ procede\ a\ trazar\ las\ gráficas\ de\ los\ siguientes\ pares\ de\ valores$:

f	
0	-1
1	3
2	13
-1	-5
-2	-15

f^{-1}	
-1	0
3	1
13	2
-5	-1
-15	-2



$En\ el\ mismo\ gráfico\ se\ marcan\ los\ puntos\ \ de\ la\ segunda\ tabla\ de\ valores\ \ \ y\ se\ tendrá\ la$ $gráfica\ de\ la\ función\ inversa\ \ f^{-1}$.

$19.-Si\ f(x) = 6 - \ x - x^3$:

$I.-Demuestre\ que\ f\ tiene\ inversa$

$II.-Calcule\ la\ pendiente\ de\ la\ recta\ tangente\ a\ la\ gráfica\ de\ f\ en\ el\ punto\ (2,-4)$.

$III.-Calcule\ la\ pendiene\ de\ la\ recta\ tangente\ a\ la\ gráfica\ de\ f^{-1}(x) en\ el\ punto\ (-4,2)$.

$IV.-Trazar\ las\ gráficas\ de\ \ f\ \ y\ \ f^{-1}\ \ en\ el\ mismo\ sistema\ de\ coordenadas$.

$Solución\ I: Dom\ F = R, \qquad f(x) = 6 - \ x - x^3 \ \Rightarrow\ f'(x) = -1 - 3x^2$

$f'(x) < 0\ \ \ \forall x \in Dom\ f,\ \ luego\ \ f\ es\ continua\ y\ monótona\ en\ Dom\ f\ \ y\ \ por\ lo\ \ tanto$ $tiene\ inversa$.

$Se\ puede\ observar\ que\ esta\ función\ tiene\ inversa,\ \ pero\ no\ podemos\ obtener\ una\ ecuación$ $que\ defina\ \ explícitamente\ la\ función\ inversa$.

Solución II: $f(x) = 6 - x - x^3 \quad \Rightarrow \; f'(x) = -1 - 3x^2$, *así que la pendiente de la recta tangente a la gráfica de* f *en* $(2, -4)$ *es* $f'(2) = -13$.

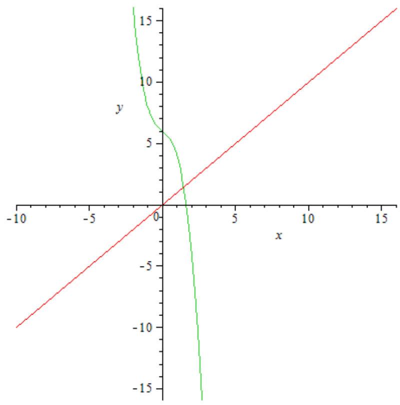
Solución III: *Pendiente de la recta tangente a la gráfica de* $f^{-1}(x)$ *en* $(-4, 2)$ *es*

$$[f^{-1}(-4)]' = \frac{1}{f'(2)} = -\frac{1}{13}$$

Solución IV : *Para trazar las gráficas de* f *y* f^{-1} *en el mismo sistema de coordenadas se procede a trazar las gráficas de los siguientes pares de valores*:

f	
0	6
1	4
2	-4
-1	8
-2	16

f^{-1}	
6	0
4	1
-4	2
8	-1
16	-2



En el mismo gráfico se marcan los puntos de la segunda tabla de valores y se tendrá la gráfica de la función inversa f^{-1}.

Demostrar que $f\left(f^{-1}(x)\right)=x$ *y que* $f^{-1}\left(f(x)\right)=x$

$1.-Si\quad f(x)=\sqrt{x}+1,\ \ determinar\ \ f^{-1}(x).\ \ \ Demostrar\ \ que\ \ f^{-1}[f(x)]=x$

$y\ \ que\ \ f[f^{-1}(x)]=x$.

$Solución.\ sea\ \ y=\sqrt{x}+1\ ,\quad entonces\quad y-1=\sqrt{x}\quad \Rightarrow\quad [y-1]^2=x$

$$La\ función\ inversa\ será\ \ f^{-1}(x)=(x-1)^2$$

$a.-Demostrar\ que\ \ f^{-1}[f(x)]=x\ ,\quad conocidas\ \begin{cases} f^{-1}(x)=(x-1)^2 \\ f(x)=\sqrt{x}+1 \end{cases}$

$$f^{-1}[f(x)]=\ [(f(x))-1]^2$$

$$f^{-1}[f(x)]=\left[\left(\sqrt{x}+1\right)-1\right]^2\quad \Rightarrow\quad f^{-1}[f(x)]=\left(\sqrt{x}\right)^2=x$$

$b.-Demostrar\ que\ \ f[f^{-1}(x)]=x\quad conocidas\ \begin{cases} f(x)=\sqrt{x}+1 \\ f^{-1}(x)=(x-1)^2 \end{cases}$

$$f[f^{-1}(x)]=\ \ \sqrt{f^{-1}(x)}+1$$

$$f[f^{-1}(x)]=\sqrt{(x-1)^2}+1\quad \Rightarrow\ \ f[f^{-1}(x)]=(x-1)+1=x$$

$2.-Si\quad f(x)=1+\left(x^{\frac{1}{2}}+2\right)^{\frac{5}{3}},\ \ determinar\ \ f^{-1}(x).\ \ \ Demostrar\ que\ f^{-1}[f(x)]=x$

$y\ \ que\ \ f[f^{-1}(x)]=x$.

$Solución:\ \ sea\ \ y=1+\left(x^{\frac{1}{2}}+2\right)^{\frac{5}{3}}\ \ entonces\quad y-1=\left(x^{\frac{1}{2}}+2\right)^{\frac{5}{3}}\ \ \Rightarrow\ \ (y-1)^{\frac{3}{5}}=\ x^{\frac{1}{2}}+2$

$$(y-1)^{\frac{3}{5}}-2=\ x^{\frac{1}{2}}\quad \Rightarrow\quad x=\left[(y-1)^{\frac{3}{5}}-2\right]^2$$

$$La\ función\ inversa\ es\ \ f^{-1}(x)=\ \left[(x-1)^{\frac{3}{5}}-2\right]^2$$

$a.-Demostrar\ que\ \ f^{-1}[f(x)]=x\ ,\quad conocidas\ \begin{cases} f^{-1}(x)=\ \left[(x-1)^{\frac{3}{5}}-2\right]^2 \\ f(x)=1+\ \left(x^{\frac{1}{2}}+2\right)^{\frac{5}{3}} \end{cases}.$

$$f^{-1}[f(x)]=\left[(f(x)-1)^{\frac{3}{5}}-2\right]^2$$

$$f^{-1}[f(x)] = \left[\left(1 + \left(x^{\frac{1}{2}} + 2\right)^{\frac{5}{3}} - 1\right)^{\frac{3}{5}} - 2\right]^2 \;\;\Rightarrow\;\; f^{-1}[f(x)] = \left(\left(\left(x^{\frac{1}{2}} + 2\right)^{\frac{5}{3}}\right)^{\frac{3}{5}} - 2\right)^2$$

$$f^{-1}[f(x)] = \left[\left(x^{\frac{1}{2}} + 2\right) - 2\right]^2 = \left(x^{\frac{1}{2}}\right)^2 = x$$

$b.-Demostrar\ que\ \ f[f^{-1}(x)] = x\ ,\quad conocidas\ \begin{cases} f(x) = 1 + \left(x^{\frac{1}{2}} + 2\right)^{\frac{5}{3}} \\ f^{-1}(x) = \left[(x-1)^{\frac{3}{5}} - 2\right]^2 \end{cases}$

$$f[f^{-1}(x)] = 1 + \left[\left(f^{-1}(x)\right)^{\frac{1}{2}} + 2\right]^{\frac{5}{3}} \;\;\Rightarrow\;\; 1 + \left[\left(\left[(x-1)^{\frac{3}{5}} - 2\right]^2\right)^{\frac{1}{2}} + 2\right]^{\frac{5}{3}}$$

$$1 + \left[\left(\left[(x-1)^{\frac{3}{5}} - 2\right]^2\right)^{\frac{1}{2}} + 2\right]^{\frac{5}{3}} = 1 + \left[(x-1)^{\frac{3}{5}} - 2 + 2\right]^{\frac{5}{3}}$$

$$f[f^{-1}(x)] = 1 + \left[(x-1)^{\frac{3}{5}}\right]^{\frac{5}{3}} \;\;\Rightarrow\;\; f[f^{-1}(x)] = 1 + (x-1) = x$$

$3.-Si\ \ f(x) = 1 + \left(x^{\frac{3}{2}} - 2\right)^{\frac{5}{3}}\ \ determinar\ \ f^{-1}(x).\ \ Demostrar\ que\ f^{-1}[f(x)] = x$

$y\ \ que\ \ f[f^{-1}(x)] = x$.

$Solución:\ \ sea\ \ y = 1 + \left(x^{\frac{3}{2}} - 2\right)^{\frac{5}{3}}\ \ entonces\ \ y - 1 = \left(x^{\frac{3}{2}} - 2\right)^{\frac{5}{3}} \;\Rightarrow\; (y-1)^{\frac{3}{5}} = x^{\frac{3}{2}} - 2$

$$(y-1)^{\frac{3}{5}} + 2 = x^{\frac{3}{2}} \;\;\Rightarrow\;\; x = \left[(y-1)^{\frac{3}{5}} + 2\right]^{\frac{2}{3}}$$

$$La\ función\ inversa\ es\ \ f^{-1}(x) = \left[(x-1)^{\frac{3}{5}} + 2\right]^{\frac{2}{3}}$$

$a.-Demostrar\ que\ \ f^{-1}[f(x)] = x,\quad conocidas\ \begin{cases} f^{-1}(x) = \left[(x-1)^{\frac{3}{5}} + 2\right]^{\frac{2}{3}} \\ f(x) = 1 + \left(x^{\frac{3}{2}} - 2\right)^{\frac{5}{3}} \end{cases}$

$$f^{-1}[f(x)] = \left[(f(x) - 1)^{\frac{3}{5}} + 2\right]^{\frac{2}{3}}$$

$$f^{-1}[f(x)]=\left[\left(1+\left(x^{\frac{3}{2}}-2\right)^{\frac{5}{3}}-1\right)^{\frac{3}{5}}+2\right]^{\frac{2}{3}} \quad \Longrightarrow \quad f^{-1}[f(x)]=\left(\left(\left(x^{\frac{3}{2}}-2\right)^{\frac{5}{3}}\right)^{\frac{3}{5}}+2\right)^{\frac{2}{3}}$$

$$f^{-1}[f(x)]= \ \left[\left(x^{\frac{3}{2}}-2\right)+2\right]^{\frac{2}{3}}=\left(x^{\frac{3}{2}}\right)^{\frac{2}{3}}=x$$

$b.-Demostrar\ que\ \ f[f^{-1}(x)]=x, \quad conocidas$ $\begin{cases} f(x)=\ 1+\left(x^{\frac{3}{2}}-2\right)^{\frac{5}{3}} \\ f^{-1}(x)=\ \left[(x-1)^{\frac{3}{5}}+2\right]^{\frac{2}{3}} \end{cases}$

$$f[f^{-1}(x)]=\ \ 1+\left[\left(f^{-1}(x)\right)^{\frac{3}{2}}-2\right]^{\frac{5}{3}}$$

$$f[f^{-1}(x)]=\ \ 1+\left[\left(\left[(x-1)^{\frac{3}{5}}+2\right]^{\frac{2}{3}}\right)^{\frac{3}{2}}-2\right]^{\frac{5}{3}}$$

$$1+\ \left[(x-1)^{\frac{3}{5}}+2-2\right]^{\frac{5}{3}}=1+\left[(x-1)^{\frac{3}{5}}\right]^{\frac{5}{3}}$$

$$f[f^{-1}(x)]=1+(x-1)\ =x$$

$4.-Si \quad f(x)=\ \sqrt{(x^3-2)^3+5} \quad determinar \quad f^{-1}(x). \quad Demostrar\ que\ f^{-1}[f(x)]=x$

$y \quad que \quad f[f^{-1}(x)]=x$.

$Solución: \quad Sea \quad y=\ \sqrt{(x^3-2)^3+5}\ , \quad entonces \quad y^2=\ (x^3-2)^3+5$

$$y^2-5=\ (x^3-2)^3 \quad \Longrightarrow \quad \sqrt[3]{y^2-5}=\ x^3-2 \ \ \Longrightarrow \ \ \sqrt[3]{y^2-5}+2=\ x^3$$

$$x=\sqrt[3]{\sqrt[3]{y^2-5}+2}$$

$$La\ función\ inversa\ es \quad f^{-1}(x)=\ \sqrt[3]{\sqrt[3]{x^2-5}+2}$$

$a.-Demostrar\ que\ \ f^{-1}[f(x)]=x \quad conocidas$ $\begin{cases} f^{-1}(x)=\ \sqrt[3]{\sqrt[3]{x^2-5}+2} \\ f(x)=\ \sqrt{(x^3-2)^3+5} \end{cases}$

$$f^{-1}[f(x)] = \sqrt[3]{\sqrt[3]{[f(x)]^2 - 5} + 2} \implies f^{-1}[f(x)] = \sqrt[3]{\sqrt[3]{\left[\sqrt{(x^3-2)^3+5}\right]^2 - 5} + 2}$$

$$f^{-1}[f(x)] = \sqrt[3]{\sqrt[3]{[(x^3-2)^3+5] - 5} + 2}$$

$$f^{-1}[f(x)] = \sqrt[3]{\sqrt[3]{(x^3-2)^3} + 2} = \sqrt[3]{(x^3-2)+2} = \sqrt[3]{(x^3)} = x$$

$b. -Demostrar\ que\ f[f^{-1}(x)] = x\ \ conocidas\ \begin{cases} f(x) = \sqrt{(x^3-2)^3+5} \\ f^{-1}(x) = \sqrt[3]{\sqrt[3]{x^2-5}+2} \end{cases}$

$$f[f^{-1}(x)] = \sqrt{\left(\left[\sqrt[3]{\sqrt[3]{x^2-5}+2}\right]^3 - 2\right)^3 + 5}$$

$$f[f^{-1}(x)] = \sqrt{\left(\left[\sqrt[3]{x^2-5}+2\right] - 2\right)^3 + 5}$$

$$f[f^{-1}(x)] = \sqrt{\left(\sqrt[3]{x^2-5}\right)^3 + 5} = \sqrt{(x^2-5)+5} = \sqrt{x^2} = x$$

$5. -Si\ \ f(x) = \sqrt{1-(x-2)^3}\ \ determinar\ \ f^{-1}(x).\ \ Demostrar\ que\ f^{-1}[f(x)] = x$

$y\ \ que\ \ f[f^{-1}(x)] = x$.

$Solución:\ \ sea\ \ y = \sqrt{1-(x-2)^3}\ \ entonces\ \ y^2 = 1-(x-2)^3$

$$y^2 - 1 = -(x-2)^3 \implies 1 - y^2 = (x-2)^3 \implies \sqrt[3]{1-y^2} = x - 2$$

$\sqrt[3]{1-y^2} + 2 = x\ \ \ \ la\ función\ inversa\ es\ \ f^{-1}(x) = \sqrt[3]{1-x^2} + 2$

$a. -Demostrar\ que\ \ f^{-1}[f(x)] = x\ \ conocidas\ \begin{cases} f^{-1}(x) = \sqrt[3]{1-x^2}+2 \\ f(x) = \sqrt{1-(x-2)^3} \end{cases}$

$$f^{-1}[f(x)] = \sqrt[3]{1 - [f(x)]^2} + 2$$

$$f^{-1}[f(x)] = \sqrt[3]{1 - \left[\sqrt{1-(x-2)^3}\right]^2} + 2 = \sqrt[3]{1 - [1-(x-2)^3]} + 2$$

$$f^{-1}[f(x)] = \sqrt[3]{1 - 1 + (x-2)^3} + 2 = \sqrt[3]{(x-2)^3} + 2 = (x-2) + 2 = x$$

$b.-$*Demostrar que* $f[f^{-1}(x)] = x$ *conocidas* $\begin{cases} f(x) = \sqrt{1-(x-2)^3} \\ f^{-1}(x) = \sqrt[3]{1-x^2} + 2 \end{cases}$

$$f[f^{-1}(x)] = \sqrt{1-(f^{-1}(x)-2)^3}$$

$$f[f^{-1}(x)] = \sqrt{1-\left(\left[\sqrt[3]{1-x^2}+2\right]-2\right)^3}$$

$$f[f^{-1}(x)] = \sqrt{1-\left(\sqrt[3]{1-x^2}+2-2\right)^3}$$

$$f[f^{-1}(x)] = \sqrt{1-\left(\sqrt[3]{1-x^2}\right)^3} = \sqrt{1-(1-x^2)} = x$$

$6.-$*Si* $f(x) = \left[\left(1+\frac{1}{x^3}\right)^5 + 1\right]^{\frac{3}{5}}$ *determinar* $f^{-1}(x)$. *Demostrar que* $f^{-1}[f(x)] = x$

y que $f[f^{-1}(x)] = x$.

$$\textit{Solución: Sea } y = \left[\left(1+\frac{1}{x^3}\right)^5 + 1\right]^{\frac{3}{5}} \quad \textit{entonces} \quad y^{\frac{5}{3}} = \left(1+\frac{1}{x^3}\right)^5 + 1$$

$$y^{\frac{5}{3}} - 1 = \left(1+\frac{1}{x^3}\right)^5 \quad \Longrightarrow \quad \sqrt[5]{y^{\frac{5}{3}}-1} = 1+\frac{1}{x^3} \quad \Longrightarrow \quad \sqrt[5]{y^{\frac{5}{3}}-1} - 1 = \frac{1}{x^3}$$

$$\frac{1}{\sqrt[5]{y^{\frac{5}{3}}-1}-1} = x^3 \quad \Longrightarrow \quad x = \frac{1}{\sqrt[3]{\sqrt[5]{y^{\frac{5}{3}}-1}-1}}$$

$$\textit{la función inversa es } f^{-1}(x) = \frac{1}{\sqrt[3]{\sqrt[5]{x^{\frac{5}{3}}-1}-1}}$$

$a)$ *Demostrar que* $f^{-1}[f(x)] = x$ *conocidas* $\begin{cases} f^{-1}(x) = \dfrac{1}{\sqrt[3]{\sqrt[5]{x^{\frac{5}{3}}-1}-1}} \\ \\ f(x) = \left[\left(1+\dfrac{1}{x^3}\right)^5 + 1\right]^{\frac{3}{5}} \end{cases}$

$$f^{-1}[f(x)]=\frac{1}{\sqrt[3]{\sqrt[5]{(f(x))^{\frac{5}{3}}-1}-1}}$$

$$f^{-1}[f(x)]=\frac{1}{\sqrt[3]{\sqrt[5]{\left(\left[\left(1+\frac{1}{x^3}\right)^5+1\right]^{\frac{3}{5}}\right)^{\frac{5}{3}}-1}-1}}$$

$$f^{-1}[f(x)]=\frac{1}{\sqrt[3]{\sqrt[5]{\left(1+\frac{1}{x^3}\right)^5+1-1}-1}}=\frac{1}{\sqrt[3]{\sqrt[5]{\left(1+\frac{1}{x^3}\right)^5}-1}}=\frac{1}{\sqrt[3]{\left(1+\frac{1}{x^3}\right)-1}}$$

$$f^{-1}[f(x)]=\frac{1}{\sqrt[3]{\frac{1}{x^3}}}=\frac{1}{\frac{1}{x}}=x$$

$b)$ *Demostrar que* $f[f^{-1}(x)]=x$ *conocidas* $\begin{cases} f(x)=\left[\left(1+\dfrac{1}{x^3}\right)^5+1\right]^{\frac{3}{5}} \\ f^{-1}(x)=\dfrac{1}{\sqrt[3]{\sqrt[5]{x^{\frac{5}{3}}-1}-1}} \end{cases}$

$$f[f^{-1}(x)]=\left[\left(1+\frac{1}{\left(f^{-1}(x)\right)^3}\right)^5+1\right]^{\frac{3}{5}}$$

$$f[f^{-1}(x)]=\left[\left(1+\frac{1}{\left(\frac{1}{\sqrt[3]{\sqrt[5]{x^{\frac{5}{3}}-1}-1}}\right)^3}\right)^5+1\right]^{\frac{3}{5}}$$

$$f[f^{-1}(x)]=\left[\left(1+\frac{1}{\frac{1}{\sqrt[5]{x^{\frac{5}{3}}-1-1}}}\right)^5+1\right]^{\frac{3}{5}}$$

$$f[f^{-1}(x)]=\left[\left(1+\sqrt[5]{x^{\frac{5}{3}}-1}-1\right)^5+1\right]^{\frac{3}{5}}=\left[\left(\sqrt[5]{x^{\frac{5}{3}}-1}\right)^5+1\right]^{\frac{3}{5}}=\left[\left(x^{\frac{5}{3}}-1\right)+1\right]^{\frac{3}{5}}$$

$$f[f^{-1}(x)]=\left[x^{\frac{5}{3}}\right]^{\frac{3}{5}}=x$$

7.$-$*Si* $f(x)=\left[\sqrt[3]{x}+1\right]^5$ *determinar* $f^{-1}(x)$. *Demostrar que* $f^{-1}[f(x)]=x$

y que $f[f^{-1}(x)]=x$.

Solución: *Sea* $y=\left[\sqrt[3]{x}+1\right]^5$ *entonces* $y^{\frac{1}{5}}=\sqrt[3]{x}+1$

$$y^{\frac{1}{5}}-1=\sqrt[3]{x} \;\;\Rightarrow\;\; \left(y^{\frac{1}{5}}-1\right)^3=x$$

$$La \;\; función \;\; inversa \;\; es \;\; f^{-1}(x)=\left(x^{\frac{1}{5}}-1\right)^3$$

a) Demostrar que $f^{-1}[f(x)]=x$ *conocidas* $\begin{cases} f^{-1}(x)=\left(x^{\frac{1}{5}}-1\right)^3 \\ f(x)=\left[\sqrt[3]{x}+1\right]^5 \end{cases}$

$$f^{-1}[f(x)]=\left((f(x))^{\frac{1}{5}}-1\right)^3 \Rightarrow f^{-1}[f(x)]=\left(\left(\left[\sqrt[3]{x}+1\right]^5\right)^{\frac{1}{5}}-1\right)^3$$

$$f^{-1}[f(x)]=\left(\left(\sqrt[3]{x}+1\right)-1\right)^3=\left(\sqrt[3]{x}\right)^3=x$$

b) Demostrar que $f[f^{-1}(x)]=x$ *conocidas* $\begin{cases} f(x)=\left[\sqrt[3]{x}+1\right]^5 \\ f^{-1}(x)=\left(x^{\frac{1}{5}}-1\right)^3 \end{cases}$

$$f[f^{-1}(x)]=\left[\sqrt[3]{(f^{-1}(x))}+1\right]^5$$

$$f[f^{-1}(x)]=\left[\sqrt[3]{\left(x^{\frac{1}{5}}-1\right)^3}+1\right]^5=\left[x^{\frac{1}{5}}-1+1\right]^5=\left[x^{\frac{1}{5}}\right]^5=x$$

$8.-Si \quad f(x) = \left[\frac{2}{3} - \sqrt{3x+1}\right]^5 \quad determinar \quad f^{-1}(x). \quad Demostrar\ que\ f^{-1}[f(x)] = x$

$y \quad que \quad f[f^{-1}(x)] = x$.

$Solución: \ Sea \quad y = \left[\frac{2}{3} - \sqrt{3x+1}\right]^5 \quad entonces \quad y^{\frac{1}{5}} = \frac{2}{3} - \sqrt{3x+1}$

$$y^{\frac{1}{5}} = \frac{2}{3} - \sqrt{3x+1} \;\Rightarrow\; \frac{2}{3} - y^{\frac{1}{5}} = \sqrt{3x+1} \;\Rightarrow\; \left(\frac{2}{3} - y^{\frac{1}{5}}\right)^2 = 3x+1$$

$$\left(\frac{2}{3} - y^{\frac{1}{5}}\right)^2 - 1 = 3x \quad \Rightarrow \quad \frac{1}{3}\left[\left(\frac{2}{3} - y^{\frac{1}{5}}\right)^2 - 1\right] = x$$

$$La\ función\ inversa\ es\ \ f^{-1}(x) = \frac{1}{3}\left[\left(\frac{2}{3} - x^{\frac{1}{5}}\right)^2 - 1\right]$$

$a)\ Demostrar\ que\ \ f^{-1}[f(x)] = x\ \ conocidas \begin{cases} f^{-1}(x) = \frac{1}{3}\left[\left(\frac{2}{3} - x^{\frac{1}{5}}\right)^2 - 1\right] \\ f(x) = \left[\frac{2}{3} - \sqrt{3x+1}\right]^5 \end{cases}$

$$f^{-1}[f(x)] = \frac{1}{3}\left[\left(\frac{2}{3} - f(x)^{\frac{1}{5}}\right)^2 - 1\right]$$

$$f^{-1}[f(x)] = \frac{1}{3}\left[\left(\frac{2}{3} - \left(\left[\frac{2}{3} - \sqrt{3x+1}\right]^5\right)^{\frac{1}{5}}\right)^2 - 1\right]$$

$$f^{-1}[f(x)] = \frac{1}{3}\left[\left(\frac{2}{3} - \frac{2}{3} + \sqrt{3x+1}\right)^2 - 1\right] = \frac{1}{3}[3x+1-1]$$

$$f^{-1}[f(x)] = \frac{1}{3}(3x) = x$$

$b)\ Demostrar\ \ que\ \ f[f^{-1}(x)] = x\ \ conocidas \begin{cases} f(x) = \left[\frac{2}{3} - \sqrt{3x+1}\right]^5 \\ f^{-1}(x) = \frac{1}{3}\left[\left(\frac{2}{3} - x^{\frac{1}{5}}\right)^2 - 1\right] \end{cases}$

$$f[f^{-1}(x)] = \left[\frac{2}{3} - \sqrt{3(f^{-1}(x)) + 1}\right]^5$$

$$f[f^{-1}(x)] = \left[\frac{2}{3} - \sqrt{3\,\frac{1}{3}\left[\left(\frac{2}{3} - x^{\frac{1}{5}}\right)^2 - 1\right] + 1}\,\right]^5$$

$$f[f^{-1}(x)] = \left[\frac{2}{3} - \sqrt{\left(\frac{2}{3} - x^{\frac{1}{5}}\right)^2 - 1 + 1}\,\right]^5 = \left[\frac{2}{3} - \sqrt{\left(\frac{2}{3} - x^{\frac{1}{5}}\right)^2}\,\right]^5$$

$$f[f^{-1}(x)] = \left[\frac{2}{3} - \frac{2}{3} + x^{\frac{1}{5}}\right]^5 = \left[x^{\frac{1}{5}}\right]^5 = x$$

9.$-$*Si* $f(x) = \dfrac{\sqrt[3]{x^{\frac{1}{3}} + 2} + 8}{\sqrt[3]{x^{\frac{1}{3}} + 2} + 9}$ *determinar* $f^{-1}(x)$. *Demostrar que* $f^{-1}[f(x)] = x$

y que $f[f^{-1}(x)] = x$.

Solución: *Sea* $y = \dfrac{\sqrt[3]{x^{\frac{1}{3}} + 2} + 8}{\sqrt[3]{x^{\frac{1}{3}} + 2} + 9}$ *entonces*

$$y\left(\sqrt[3]{x^{\frac{1}{3}} + 2} + 9\right) = \sqrt[3]{x^{\frac{1}{3}} + 2} + 8 \quad \Longrightarrow \quad y\left(\sqrt[3]{x^{\frac{1}{3}} + 2}\right) + 9y = \sqrt[3]{x^{\frac{1}{3}} + 2} + 8$$

$$y\left(\sqrt[3]{x^{\frac{1}{3}} + 2}\right) - \sqrt[3]{x^{\frac{1}{3}} + 2} = 8 - 9y \;\Longrightarrow\; \left(\sqrt[3]{x^{\frac{1}{3}} + 2}\right)(y-1) = 8 - 9y$$

$$\left(\sqrt[3]{x^{\frac{1}{3}} + 2}\right) = \frac{8-9y}{y-1} \quad \Longrightarrow \; x^{\frac{1}{3}} + 2 = \left(\frac{8-9y}{y-1}\right)^3$$

$$x^{\frac{1}{3}} = \left(\frac{8-9y}{y-1}\right)^3 - 2 \quad \Longrightarrow \quad x = \left[\left(\frac{8-9y}{y-1}\right)^3 - 2\right]^3$$

$$La\ función\ inversa\ es\ \ f^{-1}(x) = \left[\left(\frac{8-9x}{x-1}\right)^3 - 2\right]^3$$

$$a)Demostrar\ que\ \ f^{-1}[f(x)]=x\ \ conocidas\ \begin{cases} f^{-1}(x)=\left[\left(\dfrac{8-9x}{x-1}\right)^3-2\right]^3 \\ f(x)=\dfrac{\sqrt[3]{x^{\frac{1}{3}}+2}+8}{\sqrt[3]{x^{\frac{1}{3}}+2}+9} \end{cases}$$

$$f^{-1}[f(x)]=\left[\left(\frac{8-9f(x)}{f(x)-1}\right)^3-2\right]^3$$

$$f^{-1}[f(x)]=\left[\left(\frac{8-9\left(\frac{\sqrt[3]{x^{\frac{1}{3}}+2}+8}{\sqrt[3]{x^{\frac{1}{3}}+2}+9}\right)}{\frac{\sqrt[3]{x^{\frac{1}{3}}+2}+8}{\sqrt[3]{x^{\frac{1}{3}}+2}+9}-1}\right)^3-2\right]^3$$

$$f^{-1}[f(x)]=\left[\left(\frac{8\left(\sqrt[3]{x^{\frac{1}{3}}+2}+9\right)-9\left(\sqrt[3]{x^{\frac{1}{3}}+2}+8\right)}{\left(\sqrt[3]{x^{\frac{1}{3}}+2}+8\right)-\left(\sqrt[3]{x^{\frac{1}{3}}+2}+9\right)}\right)^3-2\right]^3$$

$$f^{-1}[f(x)]=\left[\left(\frac{8\left(\sqrt[3]{x^{\frac{1}{3}}+2}\right)+72-9\left(\sqrt[3]{x^{\frac{1}{3}}+2}\right)-72}{\left(\sqrt[3]{x^{\frac{1}{3}}+2}\right)+8-\left(\sqrt[3]{x^{\frac{1}{3}}+2}\right)-9}\right)^3-2\right]^3$$

$$f^{-1}[f(x)]=\left[\left(\frac{-\left(\sqrt[3]{x^{\frac{1}{3}}+2}\right)}{-1}\right)^3-2\right]^3=\left[\left(\sqrt[3]{x^{\frac{1}{3}}+2}\right)^3-2\right]^3$$

$$f^{-1}[f(x)]=\left[x^{\frac{1}{3}}+2-2\right]^3=\left[x^{\frac{1}{3}}\right]^3=x$$

$$b)Demostrar\ que\ \ f[f^{-1}(x)]=x\ \ conocidas\ \begin{cases} f(x)=\dfrac{\sqrt[3]{x^{\frac{1}{3}}+2}+8}{\sqrt[3]{x^{\frac{1}{3}}+2}+9} \\ f^{-1}(x)=\left[\left(\dfrac{8-9x}{x-1}\right)^3-2\right]^3 \end{cases}$$

$$f[f^{-1}(x)] = \frac{\sqrt[3]{(f^{-1}(x))^{\frac{1}{3}} + 2} + 8}{\sqrt[3]{(f^{-1}(x))^{\frac{1}{3}} + 2} + 9}$$

$$f[f^{-1}(x)] = \frac{\sqrt[3]{\left(\left[\left(\frac{8-9x}{x-1}\right)^3 - 2\right]^3\right)^{\frac{1}{3}} + 2} + 8}{\sqrt[3]{\left(\left[\left(\frac{8-9x}{x-1}\right)^3 - 2\right]^3\right)^{\frac{1}{3}} + 2} + 9}$$

$$f[f^{-1}(x)] = \frac{\sqrt[3]{\left(\frac{8-9x}{x-1}\right)^3 - 2 + 2} + 8}{\sqrt[3]{\left(\frac{8-9x}{x-1}\right)^3 - 2 + 2} + 9} = \frac{\sqrt[3]{\left(\frac{8-9x}{x-1}\right)^3} + 8}{\sqrt[3]{\left(\frac{8-9x}{x-1}\right)^3} + 9}$$

$$f[f^{-1}(x)] = \frac{\frac{8-9x}{x-1} + 8}{\frac{8-9x}{x-1} + 9} = \frac{8 - 9x + 8x - 8}{8 - 9x + 9x - 9} = \frac{-x}{-1} = x$$

$10.-Si \quad f(x) = e^{\sqrt[3]{x+1}} \quad determinar \quad f^{-1}(x). \quad Demostrar\ que\ f^{-1}[f(x)] = x$

$y \quad que \quad f[f^{-1}(x)] = x$.

$Solución: \ Sea \ \ y = \ e^{\sqrt[3]{x+1}} \ \ entonces \ \ \ln y = \ln\left(e^{\sqrt[3]{x+1}}\right) \ \Rightarrow \ \ln y = \sqrt[3]{x+1}$

$$\ln^3 y = x + 1 \quad \Rightarrow \quad \ln^3 y - 1 = x$$

$La\ función\ inversa\ es\ \ f^{-1}(x) = [\ln(x)]^3 - 1$

$$a.-Demostrar\ que \quad f^{-1}[f(x)] = x \quad conocidas \ \begin{cases} f^{-1}(x) = [\ln(x)]^3 - 1 \\ \quad f(x) = e^{\sqrt[3]{x+1}} \end{cases}$$

$$f^{-1}[f(x)] = \left[\ln\left(f(x)\right)\right]^3 - 1$$

$$f^{-1}[f(x)] = \left[\ln\left(e^{\sqrt[3]{x+1}}\right)\right]^3 - 1 \quad \Rightarrow \quad f^{-1}[f(x)] = \left(\sqrt[3]{x+1}\right)^3 - 1$$

$$f^{-1}[f(x)] = (x+1) - 1 = x$$

$$b.-Demostrar\ que\ \ f[f^{-1}(x)] = x \quad conocidas \ \begin{cases} \quad f(x) = e^{\sqrt[3]{x+1}} \\ f^{-1}(x) = [\ln(x)]^3 - 1 \end{cases}$$

$$f[f^{-1}(x)] = \ e^{\sqrt[3]{f^{-1}(x)+1}} \quad \Rightarrow \quad f[f^{-1}(x)] = e^{\sqrt[3]{[\ln(x)]^3 - 1 + 1}}$$

$$f[f^{-1}(x)]=e^{\sqrt[3]{[\ln(x)]^3}} \quad\Longrightarrow\quad f[f^{-1}(x)]=e^{\ln(x)}=x$$

$11.-Si \quad f(x)=5^{(\sqrt{3x+1})^3} \quad determinar \;\; f^{-1}(x). \quad Demostrar\ que\ f^{-1}[f(x)]=x$

$y \;\; que \;\; f[f^{-1}(x)]=x$.

$Solución$: $Sea \;\; y= \; 5^{(\sqrt{3x+1})^3} \;\; entonces \;\; \ln(y)= \;\left(\sqrt{3x+1}\right)^3\ln 5$

$$\frac{\ln y}{\ln 5} \;=\; \left(\sqrt{3x+1}\right)^3 \;\;\Longrightarrow\;\; \left(\frac{\ln y}{\ln 5}\right)^{\frac{2}{3}}=3x+1 \;\;\Longrightarrow\;\; \frac{1}{3}\left(\left(\frac{\ln y}{\ln 5}\right)^{\frac{2}{3}}-1\right)=x$$

$$La\ función\ inversa\ es\ \ f^{-1}(x)=\frac{1}{3}\left(\left(\frac{\ln x}{\ln 5}\right)^{\frac{2}{3}}-1\right)$$

$a)Demostrar\ que\ f^{-1}[f(x)]=x\ conocidas \quad \begin{cases} f^{-1}(x)=\dfrac{1}{3}\left(\left(\dfrac{\ln x}{\ln 5}\right)^{\frac{2}{3}}-1\right) \\ f(x)=5^{(\sqrt{3x+1})^3} \end{cases}$

$$f^{-1}[f(x)]=\frac{1}{3}\left(\left(\frac{\ln f(x)}{\ln 5}\right)^{\frac{2}{3}}-1\right)=\frac{1}{3}\left(\left(\frac{\ln 5^{(\sqrt{3x+1})^3}}{\ln 5}\right)^{\frac{2}{3}}-1\right)$$

$$f^{-1}[f(x)]=\frac{1}{3}\left(\left(\frac{\ln 5^{(3x+1)}}{\ln 5}\right)-1\right)=\frac{1}{3}\left(\left(\frac{(3x+1)\ln 5}{\ln 5}\right)-1\right)=\frac{1}{3}\left((3x+1)-1\right)$$

$$f^{-1}[f(x)]=\frac{1}{3}\left((3x+1)-1\right)=\frac{1}{3}(3x)=x$$

$b)Demostrar\ que\ f[f^{-1}(x)]=x\ \ conocidas \quad \begin{cases} f(x)=5^{(\sqrt{3x+1})^3} \\ f^{-1}(x)=\dfrac{1}{3}\left(\left(\dfrac{\ln x}{\ln 5}\right)^{\frac{2}{3}}-1\right) \end{cases}$

$$f[f^{-1}(x)]= \; f(x)=5^{\left(\sqrt{3f^{-1}(x)+1}\right)^3}$$

$$f[f^{-1}(x)]= \; 5^{\left(\sqrt{3\frac{1}{3}\left(\left(\frac{\ln x}{\ln 5}\right)^{\frac{2}{3}}-1\right)+1}\right)^3} \;=\; 5^{\left(\sqrt{\left(\left(\frac{\ln x}{\ln 5}\right)^{\frac{2}{3}}-1\right)+1}\right)^3} \;=\; 5^{\left(\sqrt{\left(\frac{\ln x}{\ln 5}\right)^{\frac{2}{3}}-1+1}\right)^3}$$

$$f[f^{-1}(x)] = \; 5^{\left(\sqrt{\left(\frac{\ln x}{\ln 5}\right)^{\frac{2}{3}} - 1 + 1}\right)^{3}} = 5^{\left(\sqrt{\left(\frac{\ln x}{\ln 5}\right)^{\frac{2}{3}}}\right)^{3}} = 5^{\frac{\ln x}{\ln 5}}$$

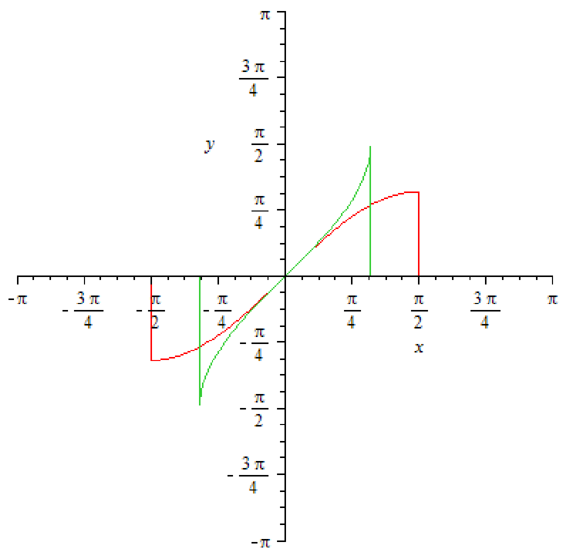
$$f[f^{-1}(x)] = 5^{\frac{\ln x}{\ln 5}} \;\; \Longrightarrow \;\; \ln(f[f^{-1}(x)]) = \frac{\ln x}{\ln 5}\ln(5) \;\; \Longrightarrow \;\; \ln(f[f^{-1}(x)]) = \; \ln{(x)}$$

$$f[f^{-1}(x)] = x$$

$GRAFICAS\ \ DE\ FUNCIONES$
$TRIGONOMETRICAS\ \ Y\ \ SUS\ \ INVERSAS$

$1.- \quad y = \operatorname{sen}(x)$

$$Gráfica\ de\ \operatorname{sen}(x)\ \ y\ \ \operatorname{arcsen}(x)$$
$$(Dominio\ restringido\ para\ asegurar\ la\ existencia\ de\ la\ inversa)$$



$$f(x) = \operatorname{sen}(x) \quad \Rightarrow \quad f'(x) = \cos(x)$$

$$\forall\, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \qquad f'(x) = \cos(x) \geq 0 \quad entonces\ \ f'(x) \geq 0\ \ en\ \ \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].$$

$$f(x)\ \ es\ \ continua\ y\ \ creciente\ \ en\ \ \left[-\frac{\pi}{2}, \frac{\pi}{2}\right],\ \ por\ lo\ tanto\ tiene\ inversa.$$

$$valor\ mínimo\ de\ f(x):\ \operatorname{sen}(x) = -1 \quad \Rightarrow \quad x = -\frac{\pi}{2} \quad , \quad A\left(-\frac{\pi}{2}, -1\right)$$

$$valor\ máximo\ de\ f(x):\quad \operatorname{sen}(x) = 1 \quad \Rightarrow \quad x = \frac{\pi}{2} \quad , \quad B\left(\frac{\pi}{2}, 1\right)$$

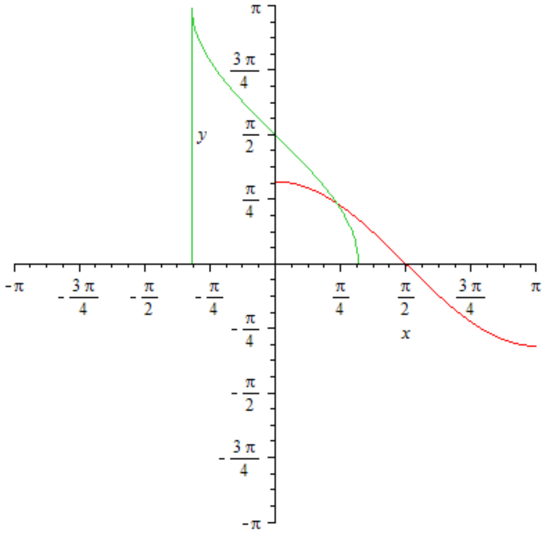
$$\operatorname{sen}(x) = 0 \quad \Rightarrow \quad x = 0\,, \quad C(0,0)$$

$$La\ inversa\ de\ f(x)\ \ está\ definida\ por\ \ f^{-1}(x) = \operatorname{arcsen} x\,.$$

$f(x) = \operatorname{sen}(x)$	$f^{-1} = \operatorname{arcsen}(x)$
$Dom\ f = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$Dom\ f^{-1} = [-1,1]$
$Rang\ f = [-1,1]$	$Rang\ f^{-1} = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$2.- \ y = \cos(x)$

$$Gráfica\ de\ \cos(x)\ y\ \arccos(x)$$
$$(Dominio\ restringido\ para\ asegurar\ la\ existencia\ de\ la\ inversa)$$



$$f(x) = \cos(x) \quad \Rightarrow \quad f'(x) = -\mathrm{sen}(x)$$

$$\forall x \in [0,\ \pi]\ es\ f'(x) = -\mathrm{sen}(x) \leq 0, \quad entonces\ f'(x) \leq 0\ en\ [0,\ \pi]$$

$$luego\ f(x)\ es\ continua\ y\ decreciente\ en\ [0,\ \pi],\ y\ por\ tanto\ tiene\ inversa$$

$$valor\ mínimo\ de\ f(x): \cos(x) = -1 \quad \Rightarrow \quad x = \pi\ ,\ A(\pi, -1)$$

$$valor\ máximo\ de\ f(x): \cos(x) = 1 \quad \Rightarrow \quad x = 0\ ,\ B(0, 1)$$

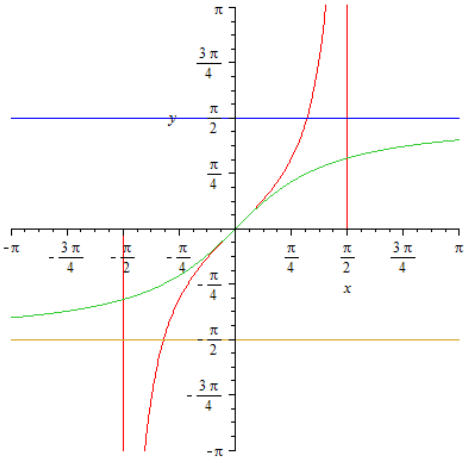
$$\cos(x) = 0 \quad \Rightarrow \quad x = \frac{\pi}{2}\ ,\quad C\left(\frac{\pi}{2}, 0\right)$$

$$f(x) = cos(x)\ \ tiene\ inversa\ \ y\ está\ definida\ por\ f^{-1}(x) = \arccos(x).$$

$f(x) = \cos(x)$	$f^{-1} = \arccos(x)$
$Dom\ f = [0,\ \pi]$	$Dom\ f^{-1} = [-1, 1]$
$Rang\ f = [-1,1]$	$Rang\ f^{-1} = [0,\ \pi]$

3.− $y = \tan(x)$

$$Gráfica\ de\ \tan(x)\ y\ \arctan(x)$$
$$(Dominio\ restringido\ para\ asegurar\ la\ existencia\ de\ la\ inversa)$$



$f(x) = \tan(x) \;\Rightarrow\; f(x) = \dfrac{\text{sen}\,(x)}{\cos\,(x)},\;\; f'(x) = \sec^2(x) > 0\;\; en\;\; \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right).$

$f(x) = \tan(x)$ *es continua y creciente en* $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ *y por tanto tiene inversa*

la cual viene dada por $f^{-1}(x) = \arctan(x)$.

$$x = \frac{\pi}{2} \;\Rightarrow\; \cos\left(\frac{\pi}{2}\right) = 0 \quad así\ que\ \tan(x)\ no\ está\ definidad\ para\ x = \frac{\pi}{2}$$

$$x = -\frac{\pi}{2} \;\Rightarrow\; \cos\left(-\frac{\pi}{2}\right) = 0 \quad así\ que\ \tan(x)\ no\ está\ definidad\ para\ x = -\frac{\pi}{2}$$

$$x = 0 \;\Rightarrow\; \cos(0) = 1 \quad luego\ \tan(0) = 0\ , \quad A(0,1)$$

$$x \in [0, \frac{\pi}{2}),\;\; x \to \left(\frac{\pi}{2}\right)^- \Rightarrow \text{sen}(x) \to 1,\;\; \cos(x) \to 0^+,\;\; \Rightarrow \lim_{x \to \left(\frac{\pi}{2}\right)^-} \tan(x) = \infty$$

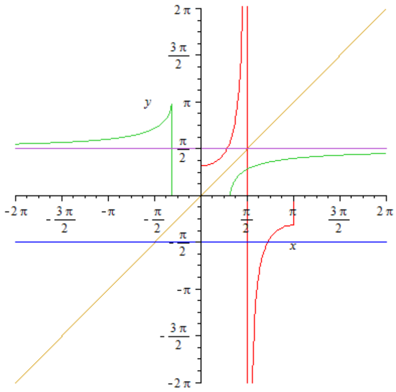
$$x \in \left(-\frac{\pi}{2}, 0\right],\;\; x \to \left(-\frac{\pi}{2}\right)^+ \Rightarrow \text{sen}(x) \to -1,\;\; \cos(x) \to 0^+,\;\; \Rightarrow \lim_{x \to \left(-\frac{\pi}{2}\right)^+} \tan(x) = -\infty$$

$$x \to \left(\frac{\pi}{2}\right)^- \;\Rightarrow\; y \to \infty,\;\; x \to \left(-\frac{\pi}{2}\right)^+ \;\Rightarrow\; y \to -\infty$$

$f(x) = \tan(x)$	$f^{-1}(x) = \arctan(x)$
$Dom\ f = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	$Dom\ f^{-1} = (-\infty, \infty)$
$Rang\ f = (-\infty, \infty)$	$Rang\ f^{-1} = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$4.- \quad y = \sec(x)$

$$Gráfica\ de\ \sec(x)\ y\ \operatorname{arcsec}(x)$$
$$(Dominio\ restringido\ para\ asegurar\ laexistencia\ de\ la\ inversa)$$



$$f(x) = \sec(x) = \frac{1}{\cos(x)}, \quad no\ está\ definida\ para\ x = \frac{\pi}{2}$$

$$Si\ \ f(x) = \sec(x)\ \ es\ \ f'(x) = \frac{\operatorname{sen} x}{\cos^2 x}, \quad \forall x \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right],$$

$$x \in \left[0, \frac{\pi}{2}\right) \Rightarrow \begin{cases} \operatorname{sen} x > 0 \\ \cos x > 0 \Rightarrow \cos^2 x > 0 \end{cases} \Rightarrow f'(x) > 0\ en\ \left[0, \frac{\pi}{2}\right)$$

$$x \in \left(\frac{\pi}{2}, \pi\right] \Rightarrow \begin{cases} \operatorname{sen} x > 0 \\ \cos x < 0 \Rightarrow \cos^2 x > 0 \end{cases} \Rightarrow f'(x) > 0\ en\ \left(\frac{\pi}{2}, \pi\right]$$

$$\sec(x)\ \ es\ continua\ y\ creciente, \ \ por\ tanto\ existe\ la\ inversa\ \ f^{-1}(x) = \operatorname{arcsec}(x).$$

$$si\ \ x = 0, \ \ entonces\ \ \frac{1}{\cos(x)} = \frac{1}{\cos(0)} = 1, \ \ así\ que\ \ \sec(0) = 1\ , \quad A(0,1)$$

$$si\ \ \ x = \pi\ , \ \ entonces\ \ \frac{1}{\cos(x)} = \frac{1}{\cos(\pi)} = -1\ , \ \ así\ que\ \ \sec(x) = -1\ , \ \ B(\pi, -1)$$

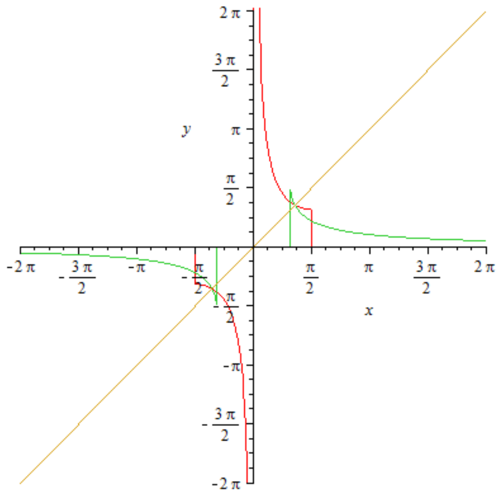
$$si\ \ x \in \left[0, \frac{\pi}{2}\right), \ \ x \to \frac{\pi^-}{2} \ \Rightarrow\ \cos(x) \to 0^+, \ \ entonces\ \lim_{x \to \frac{\pi}{2}^-} \sec(x) = \infty\ ,$$

$$si\ \ x \in \left(\frac{\pi}{2}, \pi\right], \ \ x\ \to \frac{\pi^+}{2} \ \Rightarrow\ \cos(x) \to 0^-, \ \ luego\ \ \lim_{x \to \frac{\pi}{2}^+} \sec(x) = -\infty,$$

$f(x) = \sec(x)$	$f^{-1} = \operatorname{arcsec}(x)$
$Dom\ f = \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$	$Dom\ f^{-1} = (-\infty, -1] \cup [1, \infty)$
$Rang\ f = (-\infty, -1] \cup [1, \infty)$	$Rang\ f^{-1} = \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

$5.- \; y = \csc(x)$

$$Gráfica\ de\ \csc(x)\ y\ \operatorname{arccsc}(x)$$
$$(Dominio\ restringido\ para\ asegurar\ la\ existencia\ de\ la\ inversa)$$



$$f(x) = \csc(x) = \frac{1}{\operatorname{sen}(x)},\ \ así\ que\ \csc(x)\ no\ está\ definida\ para\ \ x = 0$$

$$f(x) = \csc(x)\ \Rightarrow\ f'(x) = -\frac{\cos x}{\operatorname{sen}^2 x}\ \ en\ \ x \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$$

$$x \in \left[-\frac{\pi}{2}, 0\right)\ \Rightarrow\ \begin{cases}\operatorname{sen} x\ < 0\ \Rightarrow\ \operatorname{sen}^2 x > 0 \\ \qquad \cos x\ > 0\end{cases} \Rightarrow\ f'(x) < 0\ \ en\ \left[-\frac{\pi}{2}, 0\right)$$

$$x \in \left(0, \frac{\pi}{2}\right]\ \Rightarrow\ \begin{cases}\operatorname{sen} x\ > 0\ \Rightarrow\ \operatorname{sen}^2 x > 0 \\ \qquad \cos x\ > 0\end{cases} \Rightarrow f'(x) < 0\ \ en\ \ \left(0, \frac{\pi}{2}\right]$$

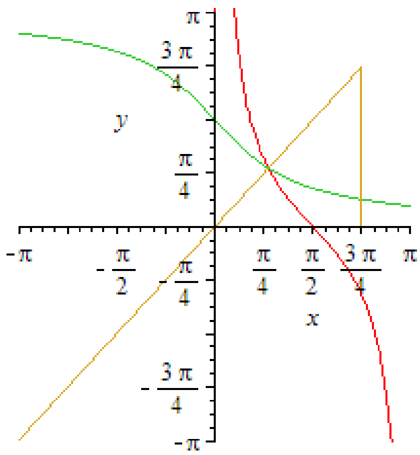
$\csc(x)$ *es continua y decreciente y por tanto existe la inversa* $f^{-1}(x) = arccsc(x)$

$$x = -\frac{\pi}{2},\ \frac{1}{\operatorname{sen}(x)}\ \ \Rightarrow\ \ \frac{1}{\operatorname{sen}\left(-\frac{\pi}{2}\right)} = -1\quad A\left(-\frac{\pi}{2}, -1\right)$$
$$x = \frac{\pi}{2},\ \frac{1}{\operatorname{sen}(x)}\ \ \Rightarrow\ \ \frac{1}{\operatorname{sen}\left(\frac{\pi}{2}\right)} = 1\quad B\left(\frac{\pi}{2}, 1\right)$$
$$\forall x \in \left(0, \frac{\pi}{2}\right],\ \operatorname{sen}(x) > 0,\ \ luego\ \lim_{x\to 0^+} \frac{1}{\operatorname{sen}(x)} = \infty \qquad x \to 0^+\ \ \Rightarrow\ \ y\ \to\ \infty$$
$$\forall x \in \left[-\frac{\pi}{2}, 0\right),\ \operatorname{sen}(x) < 0,\ \ luego\ \lim_{x\to 0^-} \frac{1}{\operatorname{sen}(x)} = -\infty \qquad x \to 0^-\ \ \Rightarrow\ \ y\ \to -\infty$$

$f(x) = \csc(x)$	$f^{-1} = \operatorname{arccsc}(x)$
$Dom\ f = \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$	$Dom\ f^{-1} = (-\infty, -1] \cup [1, \infty)$
$Rang\ f = (-\infty, -1] \cup [1, \infty)$	$Rang\ f^{-1} = \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

$6.- \ y = \ \cot(x)$

$$Gráfica\ de\ \cot(x)\ y\ \operatorname{arccot}(x)$$
$$(Dominio\ restringido\ para\ asegurar\ la\ existencia\ de\ la\ inversa)$$



$$f(x) = \cot(x) \ \Rightarrow \ f(x) = \frac{\cos(x)}{\text{sen}\ (x)},\ \ se\ tienen\ asíntotas\ \ verticales\ en\ \ x = 0\ y\ \ x = \pi$$

$$f(x) = \cot(x)\, f'(x) = -\left(\frac{1}{\text{sen}\ (x)}\right)^2 < 0,\ \ así\ que \cot(x)\ \ es\ continua\ y\ decreciente\ en$$

$$(0, \pi)\ \ \ y\ \ \ su\ inversa\ \ está\ \ definida\ por\ \ f^{-1}(x) = \operatorname{arccot}(x).$$

$$x = \frac{\pi}{2} \ \Rightarrow \ \text{sen}\left(\frac{\pi}{2}\right) = 1,\ \ \cos\left(\frac{\pi}{2}\right) = 0\ ,\ \ así\ que \cot\left(\frac{\pi}{2}\right) = 0.\ \ \ A\left(\frac{\pi}{2}, 0\right)$$

$$x = \frac{\pi}{4} \ \Rightarrow \ \text{sen}\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2},\ \ \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2},\ \ así\ que \cot\left(\frac{\pi}{4}\right) = 1.\ \ \ A\left(\frac{\pi}{4}, 1\right)$$

$$\forall x \in (0, \pi)\ ,\ \ si\ \ x \to 0^+\ \ \ entonces\ \ \ \frac{\cos(x)}{\text{sen}\ (x)} \ \to \ \ \infty$$

$$\forall x \in (0, \pi)\ ,\ \ si\ \ x \to \pi^-\ \ \ entonces\ \ \ \frac{\cos(x)}{\text{sen}(x)} \ \to \ -\infty$$

$f(x) = \cot(x)$	$f^{-1} = \operatorname{arccot}(x)$
$Dom\ f = \ \ (0, \pi)$	$Dom\ f^{-1} = \ (-\infty, \infty)$
$Rang\ f \ = \ \ (-\infty, \infty)$	$Rang\ f^{-1} = \ \ (0, \pi)$

AMPLITUD, PERIODO, DESFASE, DOMINO, RANGO,INVERSA
Y GRAFICA DE FUNCIONESTRIGONOMÉTRICAS

$En\ las\ funciones\ f(t) = A\,\mathrm{sen}[b(t-c)]\ \ y\ \ \ g(t) = A\cos[b(t-c)], los\ valores\ de\ \ A, b, y\ c$

$tienen\ los\ siguientes\ significados$:

"A" $denominada\ amplitud\ ,\ \ expresa\ el\ valor\ mínimo\ y\ \ máximo\ de\ f(t)\ \ y\ \ viene\ dada$

$por\ el\ valor\ absoluto\ de$ A, $\ es\ decir\ amplitud = \|A\|$.

$Elvalorde$ "b" $permitedeterminarelperíodo\ k\ delafunciónysecalculaporlarelación$

$k = \dfrac{2\pi}{\|b\|}$. $Este\ período\ es\ el\ numero\ real\ k\ más\ pequeño\ tal\ que\ \ f(t+k) = f(t)$.

$La\ parte\ de\ la\ gráfica\ que\ corresponde\ a\ un\ período\ se\ llama\ ciclo$.

$El\ valor\ de$ "c" $se\ llama\ desplazamiento\ o\ \ desfasamiento\ de\ la\ gráfica\ correspondiente$.

$Si\ \ c > 0\ \ la\ gráfica\ \ de\ f\ se\ desplaza\ \ c\ \ unidades\ a\ la\ izquierda\ del\ origen.\ \ Si\ \ c\ < 0$

$entonces\ el\ desplzamiento\ es\ hacia\ la\ derecha$.

$EJERCICIOS$.

$1. - Si\quad y = \ \mathrm{sen}(x)$

$I. -Determinar\ la\ amplitud,\ \ el\ período\ y\ el\ \ desfase\ de\ f$.

$II. -Hallar\ \ el\ dominio\ para\ que\ la\ función\ inversa\ exista. En\ este\ caso\ indicar\ su\ rango$.

$III. -Hallar\ la\ función\ inversa\ f^{-1}$.

$IV. -Graficar\ en\ un\ mismo\ sistema\ de\ coordenadas\ las\ funciones\ \ f\ \ y\ \ f^{-1}$.

$$Solución\ I:\quad y = \ \mathrm{sen}(x) \Rightarrow \begin{cases} amplitud\quad |A| = |1| = 1 \\ como\ b = 1\ entonces\ \ período\ de\ f:\ k = \dfrac{2\pi}{\|1\|}\ ,\ \ k = 2\pi, \\ desfase\ de\ f:\ c = 0\ , \end{cases}$$

$La\ onda\ senoide\ no\ tiene\ \ desplazamiento\ respecto\ al\ \ origen$.

$Solución\ II$: $\operatorname{sen}(x) = -1 \Rightarrow x = -\dfrac{\pi}{2}$, $\operatorname{sen}(x) = 1 \Rightarrow \mathrm{x} = \dfrac{\pi}{2}$

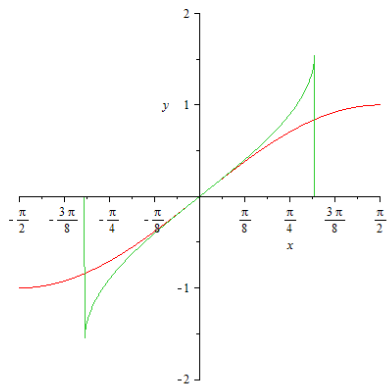
$$Para\ que\ f^{-1}\ exista\ debe\ ser\ \ Dom\ f = \left[-\frac{\pi}{2},\ \frac{\pi}{2}\right],\qquad Rang\ f = [-1,\ 1]$$

$$Solución\ III:\quad y = \operatorname{sen}(x) \Rightarrow \operatorname{arcsen} y = \operatorname{arcsen}[\operatorname{sen}(x)] \Rightarrow \operatorname{arcsen}(y) = x$$

$$Luego,\ \ la\ inversa\ de\ \ y = \operatorname{sen}(x)\ \ \ es\ \ y = \operatorname{arcsen}(x)$$

$Solución\ IV$: $Gráfica\ de\ y = \operatorname{sen}(x)\ \ y\ \ y = \operatorname{arcsen}(x)\ \ en\ el\ mismo\ sistema$ $de\ coordenadas.$

$$Gráfica\ de\ f\ \ y\ \ f^{-1}$$



$2.-$ Si $f(x) = \cos(x)$

$I.-Determinar\ la\ amplitud, el\ período\ y\ el\ desfase\ de\ f.$

$II.-Hallar\ \ el\ dominio\ para\ que\ la\ función\ inversa\ exista. En\ este\ caso\ indicar\ el\ dominio.$

$III.-Hallar\ la\ función\ inversa\ f^{-1}.$

$IV.-Graficar\ en\ un\ mismo\ sistema\ de\ coordenadas\ las\ funciones\ \ f\ \ y\ \ f^{-1}.$

$$Solución\ I: f(x) = \cos(x) \quad\Rightarrow\quad \begin{cases} amplitud\ \ |A| = |1| = 1 \\ período\ de\ f:\ k = \dfrac{2\pi}{b}\ ,\qquad k = \dfrac{2\pi}{1},\ \ k = 2\pi \\ desfase\ de\ f:\ x = 0\ , \end{cases}$$

$La\ onda\ del\ coseno\ no\ tiene\ desplazamiento\ respecto\ al\ \ origen\ .$

$$Solución\ II:\ \cos(x) = -1 \Rightarrow x = \pi\ ,\quad \cos(x) = 1 \Rightarrow x = 0$$

$$Para\ que\ f^{-1}\ exista\ debe\ ser\ \ Dom\ f = [0,\ \pi\ ],\qquad Rang\ f = [-1,\ 1]$$

$$así\ que\ Dom\ f^{-1} = [-1,\ 1]\quad y\quad Rang\ f^{-1} = [0,\ \pi]$$

$Solución\ III$:

$$y = \cos(x) \quad \Rightarrow \quad \arccos(y) = \arccos[cos(x)]$$

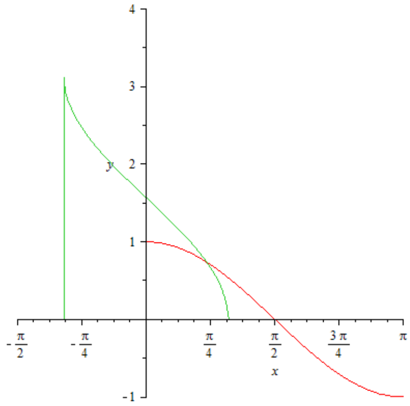
$$\arccos(y) = x \quad \Rightarrow \quad x = \arccos(y)$$

$$Así\ que\ la\ inversa\ de\ \ y = \cos(x) \quad es \quad y = \arccos(x)$$

$Solución\ IV$: $\ Gráfica\ de\ y = \cos(x)\ \ y\ \ y = \arccos(x)\ en\ el\ mismo$

$sistema\ de\ coordenadas.$

$$Gráfica\ de\ f\ \ y\ \ f^{-1}$$



3.− Si $f(x) = \cos(x + \frac{\pi}{2})$

$I.-Determinar\ la\ amplitud, el\ período\ y\ el\ desfase\ de\ f.$

$II.-Hallar\ \ el\ dominio\ para\ que\ la\ función\ inversa\ exista. En\ este\ caso\ indicar\ el\ dominio.$

$III.-Hallar\ la\ función\ inversa\ f^{-1}.$

$IV.-Graficar\ en\ un\ mismo\ sistema\ de\ coordenadas\ las\ funciones\ \ f\ \ y\ \ f^{-1}.$

$Solución\ I$:

$$\begin{cases} amplitud\ \ |A| = |1| = 1 \\ período\ de\ f:\ k = \dfrac{2\pi}{b}\ , \qquad k = \dfrac{2\pi}{1},\ \ k = 2\pi \\ desfase\ de\ f:\ x + \dfrac{\pi}{2} = 0\ ,\ \ x = -\dfrac{\pi}{2} \end{cases}$$

$La\ onda\ del\ coseno\ está\ desplazada\ -\dfrac{\pi}{2}\ \ hacia\ la\ izquierda\ del\ origen\ .$

$Solución\ II$: $\cos(x + \frac{\pi}{2}) = -1 \ \Rightarrow\ x + \frac{\pi}{2} = \pi \quad \Rightarrow \quad x = \ \frac{\pi}{2}$

$$\cos(x+\frac{\pi}{2}) = 1 \quad \Rightarrow \quad x+\frac{\pi}{2} = 0 \quad \Rightarrow \quad x = -\frac{\pi}{2}$$

$Para\ que\ f^{-1}\ exista\ debe\ ser\ \ Dom\ f\ =\ \left[-\frac{\pi}{2},\ \frac{\pi}{2}\right],\qquad Rang\ f\ =\ [-1,\ 1]$

$$así\ que\ Dom\ f^{-1} =\ [-1,\ 1]\quad y\quad Rang\ f^{-1} =\ \left[-\frac{\pi}{2},\ \frac{\pi}{2}\right]$$

$Solución\ III$:

$$y = \cos(x+\frac{\pi}{2}) \quad \Rightarrow \quad \arccos(y) = \arccos\left[\cos\left(x+\frac{\pi}{2}\right)\right]$$

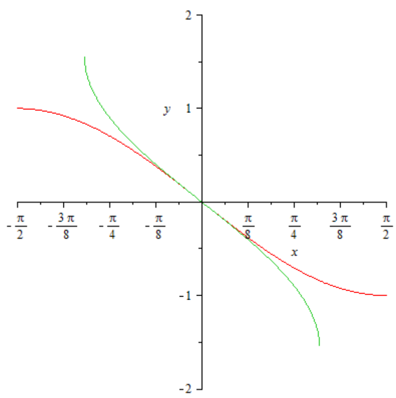
$$\arccos(y) = x+\frac{\pi}{2} \quad \Rightarrow \quad \arccos(y) - \frac{\pi}{2} = x \quad \Rightarrow \quad x = \arccos(y) - \frac{\pi}{2}$$

$$Así\ que\ la\ inversa\ de\ \ y = \cos(x+\frac{\pi}{2}) \quad es \quad y = \arccos(x) - \frac{\pi}{2}$$

$Solución\ IV$: $\ Gráfica\ de\ f(x) = \cos\left(x+\frac{\pi}{2}\right)\ \ y\ \ f^{-1}(\mathrm{x}) = \arccos(x) - \frac{\pi}{2}\quad en\ el\ mismo$

$sistema\ de\ coordenadas.$

$$Gráfica\ de\ f\ \ y\ \ f^{-1}$$



$4. - Si\qquad y =\ \mathrm{sen}\left(\frac{\pi x}{4} - \pi\right)$

$I. -Determinar\ la\ amplitud,\ \ el\ período\ y\ el\ \ desfase\ de\ f.$

$II. -Hallar\ \ el\ dominio\ para\ que\ la\ función\ inversa\ exista. En\ este\ caso\ indicar\ su\ rango.$

$III. - Hallar\ la\ función\ inversa\ f^{-1}.$

$IV. - Graficar\ en\ un\ mismo\ sistema\ de\ coordenadas\ las\ funciones\ \ f\ \ y\ \ f^{-1}.$

$Solución\ I$: $y = \operatorname{sen}\left(\frac{\pi x}{4} - \pi\right) \;\Rightarrow\; y = \operatorname{sen}\left(\frac{\pi}{4}x - \pi\right)$

$$\begin{cases} amplitud \quad |A| = |1| = 1 \\ período\ de\ f\text{:}\ k = \dfrac{2\pi}{b}\ ,\ \ k = \dfrac{2\pi}{\frac{\pi}{4}},\ \ k = 8 \\ desfase\ de\ f\text{:}\ \dfrac{\pi}{4}x - \pi = 0\ ,\ \ \dfrac{\pi}{4}x = \pi, \qquad x = 4 \end{cases}$$

$La\ onda\ senoide\ esta\ desplazada\ 4\ unidades\ hacia\ la\ derecha\ del\ origen.$

$Solución\ II$: $\operatorname{sen}\left(\frac{\pi}{4}x - \pi\right) = -1 \;\Rightarrow\; \frac{\pi}{4}x - \pi = -\frac{\pi}{2} \;\Rightarrow\; x = 2$

$$\operatorname{sen}\left(\frac{\pi}{4}x - \pi\right) = 1 \;\Rightarrow\; \frac{\pi}{4}x - \pi = \frac{\pi}{2} \;\Rightarrow\; x = 6$$

$$Para\ que\ f^{-1}\ exista\ debe\ ser\ \ Dom\ f = [2,\ 6], \qquad Rang\ f = [-1,\ 1]$$

$Solución\ III$:

$$y = \sin\left(\frac{\pi}{4}x - \pi\right) \;\Rightarrow\; \operatorname{arcsen} y = \operatorname{arcsen}\left[\operatorname{sen}\left(\frac{\pi}{4}x - \pi\right)\right] \;\Rightarrow\; \operatorname{arcsen}(y) = \frac{\pi}{4}x - \pi$$

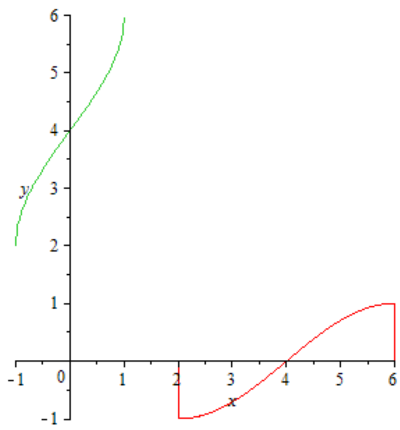
$$4\operatorname{arcsen}(y) + 4\pi = \pi x \;\Rightarrow\; x = \frac{4}{\pi}\operatorname{arcsen}(y) + 4$$

$$\text{Luego, la inversa de } y = \operatorname{sen}\left(\frac{\pi}{4}x - \pi\right)\ \ es\ \ y = \frac{4}{\pi}\operatorname{arcsen}(x) + 4$$

$Solución\ IV$: $Gráfica\ de\ f(x) = \operatorname{sen}\left(\frac{\pi x}{4} - \pi\right)\ \ y\ \ f^{-1}(x) = \frac{4}{\pi}\operatorname{arcsen}(x) + 4\ \ \ en\ el\ mismo$

$sistema\ de\ coordenadas.$

$$Gráfica\ de\ \ f\ \ y\ \ f^{-1}$$



$5.-\ Dada\ la\ función \quad y = \tan(2x + \frac{\pi}{3})$

$I.-Determinar\ la\ amplitud,\ \ el\ período\ y\ el\ \ desfase\ de\ f.$

$II.-Hallar\ \ el\ dominio\ para\ que\ la\ función\ inversa\ exista.En\ este\ caso\ indicar\ su\ rango.$

$III.-Hallar\ la\ función\ inversa\ f^{-1}.$

$IV.-Graficar\ en\ un\ mismo\ sistema\ de\ coordenadas\ las\ funciones\ \ f\ \ y\ \ f^{-1}.$

$Solución\ I:\ \ Para \quad y = \ \tan(2x + \frac{\pi}{3})$

$$\begin{cases} amplitud \quad |A| = \infty \\ período\ de\ f:\ \ k = \frac{\pi}{b}\ ,\ \ k = \frac{\pi}{2} \\ desfase\ de\ f:\ \ 2x + \frac{\pi}{3} = 0\ ,\ \ 2\mathrm{x} = -\frac{\pi}{3} \ \ \Rightarrow \ \ x = -\frac{\pi}{6} \end{cases}$$

$La \quad gráfica\ de\ \ \tan(x)\ \ está\ \ desplazada\ x = -\frac{\pi}{6}\ unidades\ hacia\ la\ izquierda\ del\ origen.$

$Solución \ \ II:-Hallar\ \ el\ dominio\ de\ \ f\ \ para\ que \ \ f^{-1}\ \ inversa\ exista.$

$$\tan(2x + \frac{\pi}{3}) = \ \tan(\frac{\pi}{2}) = \ \infty \quad \Rightarrow \ 2x + \frac{\pi}{3} = \ \frac{\pi}{2} \quad \Rightarrow \quad x = \frac{\pi}{12}$$

$$\tan(2x + \frac{\pi}{3}) = \ \tan\left(-\frac{\pi}{2}\right) = \ \infty \quad \Rightarrow \ 2x + \frac{\pi}{3} = -\ \frac{\pi}{2} \quad \Rightarrow \quad x = -\frac{5\pi}{12}$$

$$Dom\ f = \left(-\frac{5\pi}{12}, \frac{\pi}{12}\right) \qquad Rang\ f = \ (-\infty, \infty)$$

$Solución\ III$: $y = \tan(2x + \frac{\pi}{3}) \quad \Rightarrow \quad \arctan y = \arctan\left[\tan\left(2x + \frac{\pi}{3}\right)\right]$

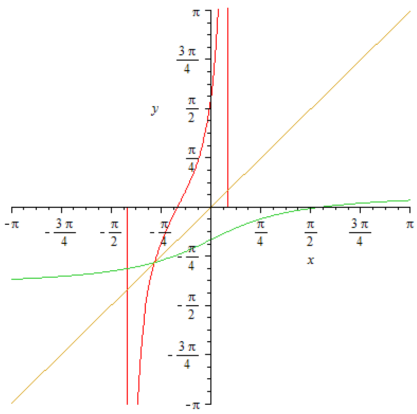
$$\arctan y = 2x + \frac{\pi}{3} \quad \Rightarrow \quad \arctan y - \frac{\pi}{3} = 2x \quad \Rightarrow \quad \frac{1}{2}\arctan y - \frac{\pi}{6} = x$$

$$\text{Así, la inversa de } \ y = \tan(2x + \frac{\pi}{3}) \quad es \quad y = \frac{1}{2}\arctan x - \frac{\pi}{6}$$

$$Dom\ f^{-1} = \ (-\infty, \infty), \quad Rang\ f^{-1} = \left(-\frac{5\pi}{12}, \frac{\pi}{12}\right)$$

$Solución\ IV$: $Gráfica\ de\ f(x) = \ \tan\left(2x + \frac{\pi}{3}\right)\ \ y\ \ f^{-1}(x) = \frac{1}{2}\arctan x - \frac{\pi}{6}$.

$$Gráfica\ de\ \ f\ \ y\ \ f^{-1}$$



$6.-$ Si $\quad y = \frac{3}{4} + \text{sen}\left(\frac{3x}{2} - \frac{\pi}{6}\right)$

$I.-Determinar\ la\ amplitud,\ \ el\ período\ y\ el\ \ desfase\ de\ f.$

$II.-Hallar\ \ el\ dominio\ para\ que\ la\ función\ inversa\ exista. En\ este\ caso\ indicar\ su\ rango.$

$III.-Hallar\ la\ función\ inversa\ f^{-1}.$

$IV.-Graficar\ en\ un\ mismo\ sistema\ de\ coordenadas\ las\ funciones\ \ f\ \ y\ \ f^{-1}.$

$Solución\ I : Si$ Si $\quad y = \frac{3}{4} + \text{sen}\left(\frac{3x}{2} - \frac{\pi}{6}\right)$

$$\begin{cases} amplitud \quad |A| = |1| = 1 \\ período\ de\ f\text{:}\ k = \frac{2\pi}{\frac{3}{2}}\ ,\ \ k = \frac{4\pi}{3}. \\ desfase\ de\ f\text{:}\ \frac{3x}{2} - \frac{\pi}{6} = 0\ ,\ \ \frac{3}{2}x = \frac{\pi}{6},\quad x = \frac{\pi}{9} \end{cases}$$

$La\ \ gráfica\ de\ \ f\ \ está\ \ desplazada\ x = \frac{\pi}{9}\ unidades\ hacia\ la\ derecha\ \ del\ origen$

$Solución\ II\text{:}\ -Hallar\ \ el\ dominio\ de\ f\ para\ \ que\ \ f^{-1}\ \ exista.$

$$\frac{3}{4} + \text{sen}\left(\frac{3x}{2} - \frac{\pi}{6}\right) = \frac{3}{4} - 1 \quad \Rightarrow \quad \frac{3}{4} + \text{sen}\left(\frac{3x}{2} - \frac{\pi}{6}\right) = -\frac{1}{4}$$

$$\text{sen}\left(\frac{3x}{2} - \frac{\pi}{6}\right) = \text{sen}\left(-\frac{\pi}{2}\right)$$

$$\frac{3x}{2} - \frac{\pi}{6} = -\frac{\pi}{2} \quad \Rightarrow \quad \frac{3x}{2} = -\frac{\pi}{2} + \frac{\pi}{6} \quad \Rightarrow \quad \frac{3x}{2} = -\frac{2\pi}{6} \quad \Rightarrow \quad x = -\frac{2\pi}{9}$$

$$\frac{3}{4} + \text{sen}\left(\frac{3x}{2} - \frac{\pi}{6}\right) = \frac{3}{4} + 1 \quad \Rightarrow \quad \frac{3}{4} + \text{sen}\left(\frac{3x}{2} - \frac{\pi}{6}\right) = \frac{7}{4}$$

$$\text{sen}\left(\frac{3x}{2} - \frac{\pi}{6}\right) = \text{sen}\left(\frac{\pi}{2}\right)$$

$$\frac{3x}{2} - \frac{\pi}{6} = \frac{\pi}{2} \quad \Rightarrow \quad \frac{3x}{2} = \frac{\pi}{2} + \frac{\pi}{6} \quad \Rightarrow \quad \frac{3x}{2} = \frac{4\pi}{6} \quad \Rightarrow \quad x = \frac{4\pi}{9}$$

$$Dom\ f = \left[-\frac{2\pi}{9}, \frac{4\pi}{9}\right] \qquad Rang = \left[-\frac{1}{4},\ \frac{7}{4}\right]$$

$Solución\ III\text{:}\ \ Hallar\ la\ función\ inversa\ f^{-1}$

$$y = \frac{3}{4} + \text{sen}\left(\frac{3x}{2} - \frac{\pi}{6}\right) \quad \Rightarrow \ y - \frac{3}{4} = \text{sen}\left(\frac{3x}{2} - \frac{\pi}{6}\right)$$

$$\text{arcsen}\left(y - \frac{3}{4}\right) = \text{arcsen}\left[\text{sen}\left(\frac{3x}{2} - \frac{\pi}{6}\right)\right] \quad \Rightarrow \quad \text{arcsen}\left(y - \frac{3}{4}\right) = \frac{3x}{2} - \frac{\pi}{6}$$

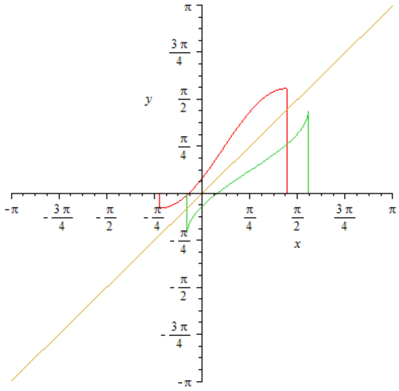
$$\text{arcsen}\left(y - \frac{3}{4}\right) + \frac{\pi}{6} = \frac{3}{2}x \quad \Rightarrow \quad \frac{2}{3}\text{arcsen}\left(y - \frac{3}{4}\right) + \frac{\pi}{9} = x$$

$$La\ inversa\ de\ \ y = \frac{3}{4} + \text{sen}(\frac{3x}{2} - \frac{\pi}{6}) \quad es \quad y = \frac{\pi}{9} + \frac{2}{3}\text{arcsen}(x - \frac{3}{4})$$

$$Dom\ f^{-1} = \left[-\frac{1}{4},\ \frac{7}{4}\right]\ \ ,\ \ Rang\ f^{-1} = \left[-\frac{2\pi}{9}, \frac{4\pi}{9}\right]$$

$Solución\ IV$: $\quad f(x) = y = \frac{3}{4} + \text{sen}\left(\frac{3x}{2} - \frac{\pi}{6}\right) \quad y \quad f^{-1}(x) = \frac{\pi}{9} + \frac{2}{3}\text{arcsen}(x - \frac{3}{4})$

$$Gráfica\ de\ \ f\ \ y\ \ de\ \ f^{-1}$$



$7. - Dada\ la\ \ función\ \ y - 3 = cos\left(2x - \frac{\pi}{3}\right)$

$I. -Determinar\ la\ amplitud,\ \ el\ período\ y\ el\ \ desfase\ de\ f.$

$II. -Hallar\ \ el\ dominio\ para\ que\ la\ función\ inversa\ exista. En\ este\ caso\ indicar\ su\ rango.$

$III. -Hallar\ la\ función\ inversa\ f^{-1}.$

$IV. -Graficar\ en\ un\ mismo\ sistema\ de\ coordenadas\ las\ funciones\ \ f\ \ y\ \ f^{-1}.$

$Solución\ I: y - 3 = \cos\left(2x - \frac{\pi}{3}\right) \quad \Rightarrow \quad y = 3 + \cos\left(2x - \frac{\pi}{3}\right)$

$$\begin{cases} amplitud \quad |A| = |1| = 1 \\ período\ de\ f\!:\ k = \dfrac{2\pi}{2}\ ,\ \ k = \pi. \\ desfase\ de\ f\!:\ 2x - \dfrac{\pi}{3} = 0\ ,\ \ 2x = \dfrac{\pi}{3},\ \ \ x = \dfrac{\pi}{6} \end{cases}$$

$La\ onda\ del\ coseno\ está\ desplazada\ x = \frac{\pi}{6}\ \ \ unidades\ \ hacia\ la\ derecha\ del\ origen.$

$Solución\ II. -Hallar\ \ el\ dominio\ de\ f\ \ para\ que\ \ f^{-1}\ \ exista.$

$$3 + \cos\left(2x - \frac{\pi}{3}\right) = 3 - 1 = 2$$

$$\cos\left(2x - \frac{\pi}{3}\right) = \cos(\pi) \quad \Rightarrow \quad 2x - \frac{\pi}{3} = \pi \quad \Rightarrow \quad x = \frac{4\pi}{6}\ ,\ \ x = \frac{2\pi}{3}$$

$$3+\cos\left(2x-\frac{\pi}{3}\right)=3+1=4$$

$$\cos\left(2x-\frac{\pi}{3}\right)=\cos(0) \quad \Rightarrow \quad 2x-\frac{\pi}{3}=0 \quad \Rightarrow \quad 2x=\frac{\pi}{3}\ ,\ \ x=\frac{\pi}{6}$$

$$Dom\ \ f=\left[\frac{\pi}{6},\frac{2\pi}{3}\right] \qquad Rang\ f=\ [2,4]$$

$Solución\ III.-Hallar\ la\ función\ inversa\ f^{-1}.$

$$y-3=\cos\left(2x-\frac{\pi}{3}\right) \quad \Rightarrow \quad \arccos(y-3)=\arccos\left[\cos\left(2x-\frac{\pi}{3}\right)\right]$$

$$\arccos(y-3)=\ 2x-\frac{\pi}{3} \qquad \Rightarrow \quad \frac{\pi}{3}+\arccos(y-3)=2x$$

$$\frac{\pi}{6}+\frac{1}{2}\arccos(y-3)=x$$

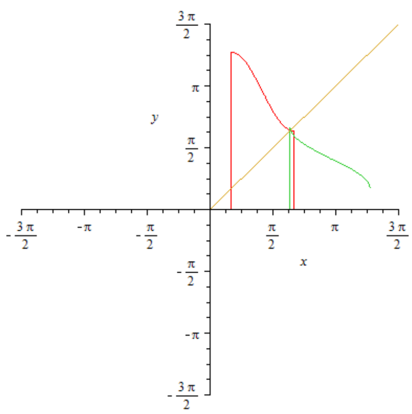
$$La\ inversa\ de\quad y-3=\cos\left(2x-\frac{\pi}{3}\right)\quad es\ \ y=\frac{\pi}{6}+\frac{1}{2}\arccos(\mathrm{x}-3)$$

$$Dom\ \ f^{-1}=\ [2,4]\ \ ,\ \ Rang\ \ f^{-1}=\left[\frac{\pi}{6},\frac{2\pi}{3}\right]$$

$Solución\ IV: Graficar\ en\ un\ mismo\ sistema\ de\ coordenadas\ y-3=\cos\left(2x-\frac{\pi}{3}\right)$

$y=\frac{\pi}{6}+\frac{1}{2}\arccos(x-3).$

$$Gráfica\ de\ \ f\ \ y\ \ de\ \ f^{-1}$$



$8. - \textit{Dada la función} \quad y = 1 + \operatorname{sen}\left(\frac{\pi x}{3}\right)$

$I. - \textit{Determinar la amplitud, el período y el desfase de } f.$

$II. - \textit{Hallar el dominio para que la función inversa exista. En este caso indicar su rango.}$

$III. - \textit{Hallar la función inversa } f^{-1}.$

$IV. - \textit{Graficar en un mismo sistema de coordenadas las funciones } f \textit{ y } f^{-1}.$

$\textit{Solución } I\text{:} \; y = 1 + \operatorname{sen}\left(\frac{\pi x}{3}\right)$

$$\begin{cases} \textit{amplitud} \quad |A| = |1| = 1 \\ \textit{período de } f\text{:} \; k = \dfrac{2\pi}{\frac{\pi}{3}} \;, \; k = 6. \\ \textit{desfase de } f\text{:} \; x = 0 \end{cases}$$

$\textit{La onda de} \quad y = 1 + \operatorname{sen}\left(\frac{\pi x}{3}\right) \quad \textit{no está desplazada respecto del origen.}$

$\textit{Solución } II. - \textit{Hallar el dominio para que la función inversa exista.}$

$$1 + \operatorname{sen}\left(\frac{\pi x}{3}\right) = 1 - 1 = 0$$

$$\operatorname{sen}\left(\frac{\pi x}{3}\right) = \operatorname{sen}\left(-\frac{\pi}{2}\right) \;\Rightarrow\; \frac{\pi x}{3} = -\frac{\pi}{2} \quad x = -\frac{3}{2}$$

$$1 + \operatorname{sen}\left(\frac{\pi x}{3}\right) = 1 + 1 = 2$$

$$\operatorname{sen}\left(\frac{\pi x}{3}\right) = \operatorname{sen}\left(\frac{\pi}{2}\right) \;\Rightarrow\; \frac{\pi x}{3} = \frac{\pi}{2} \quad x = \frac{3}{2}$$

$$\textit{Dom } f = \left[-\frac{3}{2}, \frac{3}{2}\right] \;, \quad \textit{Rang } f = [0,2]$$

$\textit{Solución } III. - \textit{Hallar la función inversa } f^{-1}.$

$$y = 1 + \operatorname{sen}\left(\frac{\pi x}{3}\right) \;\Rightarrow\; y - 1 = \operatorname{sen}\left(\frac{\pi x}{3}\right) \;\Rightarrow\; \operatorname{arcsen}(y-1) = \operatorname{arcsen}\left[\operatorname{sen}\left(\frac{\pi x}{3}\right)\right]$$

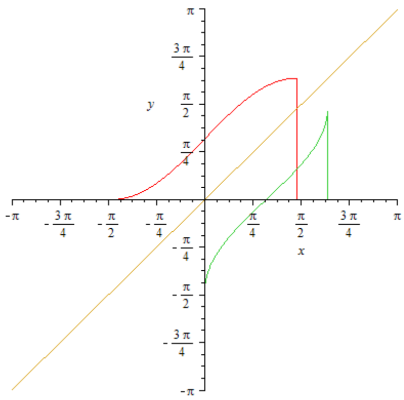
$$\text{arcsen}(y-1)=\frac{\pi x}{3} \quad \Rightarrow \quad \frac{3}{\pi}\text{arcsen}(y-1)=x$$

$$\text{Así que la inversa de } \; y=1+\text{sen}\left(\frac{\pi x}{3}\right) \text{ es } \; y=\frac{3}{\pi}\text{arcsen}(x-1)$$

$$Dom\ f^{-1}=\ [0,2]\ ,\quad Rang\ \ f=\left[-\frac{3}{2},\frac{3}{2}\right]$$

$Solución\ \ IV:$

$$Gráfica\ de\ \ f\ \ y\ \ de\ \ f^{-1}$$



$9.-Dada\ \ la\ función\ \ y=2+3\cos\left(\frac{\pi x}{2}-\pi\right)$

$I.-Determinar\ la\ amplitud,\ \ el\ período\ y\ el\ \ desfase\ de\ f.$

$II.-Hallar\ \ el\ dominio\ para\ que\ la\ función\ inversa\ exista.\ En\ este\ caso\ indicar\ su\ rango.$

$III.-Hallar\ la\ función\ inversa\ f^{-1}.$

$IV.-Graficar\ en\ un\ mismo\ sistema\ de\ coordenadas\ las\ funciones\ \ f\ \ y\ \ f^{-1}.$

$$Solución\ I:\quad y=2+3\cos\left(\frac{\pi x}{2}-\pi\right) \quad \Rightarrow \quad \begin{cases} amplitud \quad |A|=|3|=3 \\ período\ de\ f:\ k=\dfrac{2\pi}{\frac{\pi}{2}}\ ,\ \ k=4. \\ desfase\ de\ f:\ \dfrac{\pi x}{2}-\pi=0 \ \Rightarrow \ \ x=2 \end{cases}$$

$La\ onda\ coseno\ esta\ desplazada\ 2\ unidades\ hacia\ la\ derecha\ del\ origen$

$Solución\ II:\ \ Dominio\ para\ que\ la\ función\ inversa\ exista.$

$$2+3\cos\left(\frac{\pi x}{2}-\pi\right)=2-3 \ \Rightarrow \ \ 2+\cos\left(\frac{\pi x}{2}-\pi\right)=-1$$

$$3\cos\left(\frac{\pi x}{2}-\pi\right)=-3 \;\Rightarrow\; \cos\left(\frac{\pi x}{2}-\pi\right)=-1 \;\Rightarrow\; \cos\left(\frac{\pi x}{2}-\pi\right)=\cos(\pi)$$

$$\frac{\pi x}{2}-\pi=\pi \;\Rightarrow\; \frac{\pi x}{2}=2\pi \;\Rightarrow\; x=4$$

$$2+3\cos\left(\frac{\pi x}{2}-\pi\right)=2+3 \;\Rightarrow\; 2+\cos\left(\frac{\pi x}{2}-\pi\right)=5$$

$$3\cos\left(\frac{\pi x}{2}-\pi\right)=3 \;\Rightarrow\; \cos\left(\frac{\pi x}{2}-\pi\right)=1 \;\Rightarrow\; \cos\left(\frac{\pi x}{2}-\pi\right)=\cos\,(0)$$

$$\frac{\pi x}{2}-\pi=0 \;\Rightarrow\; \frac{x}{2}=1 \;\Rightarrow\; x=2$$

$$Dom\ f=[2{,}4]\,,\ \ Rang\ f=\ [-1,5]$$

$Solución\ III.-Hallar\ la\ función\ inversa\ f^{-1}.$

$$y=2+3\cos\left(\frac{\pi x}{2}-\pi\right) \;\Rightarrow\; y-2=3\cos\left(\frac{\pi x}{2}-\pi\right) \;\Rightarrow\; \frac{1}{3}(y-2)=\cos\left(\frac{\pi x}{2}-\pi\right)$$

$$\arccos\frac{1}{3}(y-2)=\frac{\pi x}{2}-\pi \;\Rightarrow\; \arccos\frac{1}{3}(y-2)+\pi=\frac{\pi x}{2}$$

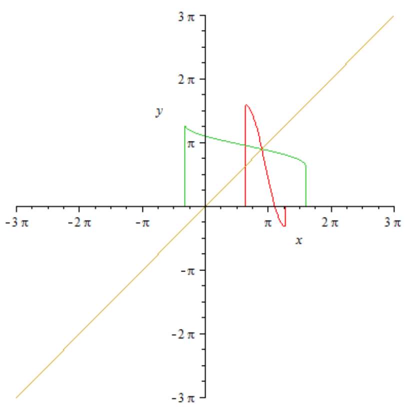
$$\frac{2}{\pi}\arccos\frac{1}{3}(y-2)+2=x$$

$$Así,\ \ la\ inversa\ de\ \ f(x)=2+3\cos\left(\frac{\pi x}{2}-\pi\right)\ \ es\ \ f^{-1}(x)=\frac{2}{\pi}\arccos\frac{1}{3}(x-2)+2$$

$$Dom\ f^{-1}=\ [-1,5]\,,\quad Rang\ f^{-1}(x)\ =[2{,}4]$$

$Solución\ \ IV\!:\ Gráfica\ \ de\ \ f(x)=2+3\cos\left(\frac{\pi x}{2}-\pi\right)\ \ y\ \ f^{-1}(x)=\frac{2}{\pi}\arccos\frac{1}{3}(x-2)+2$

$$Gráfica\ de\ \ f\ \ \ y\ \ \ f^{-1}$$



10. $-$ Dada la función $y = 1 + 3\tan\left(\dfrac{x+\pi}{6}\right)$

$I.-Determinar\ la\ amplitud,\ \ el\ período\ y\ el\ \ desfase\ de\ f.$

$II.-Hallar\ \ el\ dominio\ para\ que\ la\ función\ inversa\ exista. En\ este\ caso\ indicar\ su\ rango.$

$III.-Hallar\ la\ función\ inversa\ f^{-1}.$

$IV.-Graficar\ en\ un\ mismo\ sistema\ de\ coordenadas\ las\ funciones\ \ f\ \ y\ \ f^{-1}.$

$$Solución\ I:\quad y = 1 + 3\tan\left(\frac{x+\pi}{6}\right) \Rightarrow \begin{cases} amplitud\ \ :\ \infty \\ período\ de\ f:\ k = \dfrac{\pi}{6}.k = 6\pi \\ desfase\ de\ f:\ \dfrac{x+\pi}{6} = 0 \ \Rightarrow\ x = -\pi \end{cases}$$

$La\ onda\ tangente\ esta\ desplazada - \pi\ \ unidades\ hacia\ la\ izquierda\ del\ origen.$

$Solución\ II.-Hallar\ \ el\ dominio\ para\ que\ la\ función\ inversa\ exista.$

$$1 + 3\tan\left(\frac{x+\pi}{6}\right) = 1 + 3\tan\left(\frac{\pi}{2}\right) = \infty \quad\Rightarrow\quad 3\tan\left(\frac{x+\pi}{6}\right) = 3\tan\left(\frac{\pi}{2}\right)$$

$$\frac{x+\pi}{6} = \frac{\pi}{2} \quad\Rightarrow\quad \frac{x}{6} = \frac{\pi}{2} - \frac{\pi}{6} \Rightarrow \quad \frac{x}{6} = \frac{2\pi}{6} \quad x = 2\pi$$

$$1 + 3\tan\left(\frac{x+\pi}{6}\right) = 1 + 3\tan\left(-\frac{\pi}{2}\right) = -\infty \quad\Rightarrow\quad 3\tan\left(\frac{x+\pi}{6}\right) = 3\tan\left(-\frac{\pi}{2}\right)$$

$$\frac{x+\pi}{6} = -\frac{\pi}{2} \quad\Rightarrow\quad x + \pi = -3\pi \quad\Rightarrow\quad x = -4\pi$$

$$Dom\ f = (-4\pi, 2\pi)\,,\quad Rang\ f = (-\infty, \infty)$$

$Solución$: $III.-Hallar\ la\ función\ inversa\ f^{-1}.$

$$y = 1 + 3\tan\left(\frac{x+\pi}{6}\right) \quad\Rightarrow\quad y - 1 = 3\tan\left(\frac{x+\pi}{6}\right) \quad\Rightarrow\quad \frac{1}{3}(y-1) = \tan\left(\frac{x+\pi}{6}\right)$$

$$\arctan\frac{1}{3}(y-1) = \text{artan}\left[\tan\left(\frac{x+\pi}{6}\right)\right] \quad\Rightarrow\quad \arctan\frac{1}{3}(y-1) = \frac{x+\pi}{6}$$

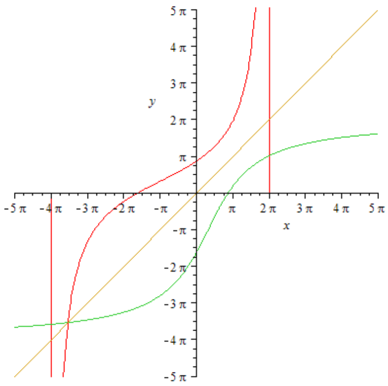
$$6\arctan\frac{1}{3}(y-1) = x + \pi \quad\Rightarrow\quad 6\arctan\frac{1}{3}(y-1) - \pi = x$$

$La\ inversa\ de\ \ y = 1 + 3\tan(\dfrac{x+\pi}{6})\quad es\quad y = 6\arctan\dfrac{1}{3}(x-1) - \pi$

$IV.-Graficar\ en\ un\ mismo\ sistema\ de\ coordenadas\ las\ funciones\ y = 1 + 3\tan\left(\frac{x+\pi}{6}\right)$

$y \qquad y = 6\arctan\frac{1}{3}(x-1) - \pi$

$Gráfica\ de\ \ f\ \ \ y\ \ \ f^{-1}$



$REGLAS\ DE\ DERIVACIÓN$

$y = k\ (constante) \quad \Rightarrow \dfrac{dy}{dx} = 0$ *La derivada de una constante es igual a cero.*

$y = x \quad \Rightarrow \dfrac{dy}{dx} = 1$ *La derivada de la variable independiente es igual a uno.*

$y = a \cdot u \quad \Rightarrow \dfrac{dy}{dx} = a\dfrac{du}{dx}$ *La derivada del producto de una constante por una función es igual al producto de la constante por la derivada de la función.*

$y = u + u + w \quad \Rightarrow \dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx} + \dfrac{dw}{dx}$ *La derivada de una suma de funciones es igual a la suma de las derivadas de las funciones.*

$y = u \cdot v \quad \Rightarrow \dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$, *derivada de un producto de funciones.*

$y = \dfrac{u}{v} \quad \Rightarrow \dfrac{dy}{dx} = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$, *derivada de un cociente de funciones.*

$y = u^n \quad \Rightarrow \dfrac{dy}{dx} = nu^{n-1}\dfrac{du}{dx}$ $\qquad y = x^n \quad \Rightarrow \dfrac{dy}{dx} = nx^{n-1}$

$y = \ln u \quad \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{u}\dfrac{du}{dx}$ $\qquad y = \ln\ x \quad \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{x}$

$y = \log u \quad \Rightarrow \dfrac{dy}{dx} = \dfrac{\log(e)}{u}\dfrac{du}{dx}$ $\qquad y = log\ x \quad \Rightarrow \dfrac{dy}{dx} = \dfrac{\log(e)}{x}$

$y = e^u \quad \Rightarrow \dfrac{dy}{dx} = e^u\dfrac{du}{dx}$ $\qquad y = e^x \quad \Rightarrow \dfrac{dy}{dx} = e^x$

$y = a^u \quad \Rightarrow \dfrac{dy}{dx} = a^u\ln(a)\dfrac{du}{dx}$ $\qquad y = a^x \quad \Rightarrow \dfrac{dy}{dx} = a^x\ln(a)$

$y = \sqrt{u} \quad \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2\sqrt{u}}\dfrac{du}{dx}$ $\qquad y = \sqrt{x} \quad \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}}$

$Derivadas\ de\ funciones\ trigonométricas$

$y = \operatorname{sen} u \Rightarrow \dfrac{dy}{dx} = \cos u\dfrac{du}{dx}$ $\qquad y = \operatorname{sen} x \Rightarrow \dfrac{dy}{dx} = \cos x$

$y = \cos u \Rightarrow \dfrac{dy}{dx} = -\operatorname{sen} u\dfrac{du}{dx}$ $\qquad y = \cos x \Rightarrow \dfrac{dy}{dx} = -\operatorname{sen} x$

$y = \tan u \Rightarrow \dfrac{dy}{dx} = \sec^2 u\dfrac{du}{dx}$ $\qquad y = \tan x \Rightarrow \dfrac{dy}{dx} = \sec^2 x$

$y = \cot u \ \Rightarrow\ \dfrac{dy}{dx} = -\csc^2 u \dfrac{du}{dx}$ $\qquad y = \cot x \ \Rightarrow\ \dfrac{dy}{dx} = -\csc^2 x$

$y = \sec u \ \Rightarrow\ \dfrac{dy}{dx} = \sec u \tan u \dfrac{du}{dx}$ $\qquad y = \sec x \ \Rightarrow\ \dfrac{dy}{dx} = \sec x \tan u\, x$

$y = \csc u \ \Rightarrow\ \dfrac{dy}{dx} = -\csc u \cot u \dfrac{du}{dx}$ $\qquad y = \csc x \ \Rightarrow\ \dfrac{dy}{dx} = -\csc x \cot x$

$y = \operatorname{arcsen} u \ \Rightarrow\ \dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}$ $\qquad y = \operatorname{arcsen} x \ \Rightarrow\ \dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

$y = \arccos u \ \Rightarrow\ \dfrac{dy}{dx} = -\dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}$ $\qquad y = \arccos x \ \Rightarrow\ \dfrac{dy}{dx} = -\dfrac{1}{\sqrt{1-x^2}}$

$y = \arctan u \ \Rightarrow\ \dfrac{dy}{dx} = \dfrac{1}{1+u^2}\dfrac{du}{dx}$ $\qquad y = \arctan x \ \Rightarrow\ \dfrac{dy}{dx} = \dfrac{1}{1+x^2}$

$y = \operatorname{arccot} u \ \Rightarrow\ \dfrac{dy}{dx} = -\dfrac{1}{1+u^2}\dfrac{du}{dx}$ $\qquad y = \operatorname{arccot} x \ \Rightarrow\ \dfrac{dy}{dx} = -\dfrac{1}{1+x^2}$

$y = \operatorname{arcsec} u \ \Rightarrow\ \dfrac{dy}{dx} = \dfrac{1}{u\sqrt{u^2-1}}\dfrac{du}{dx}$ $\qquad y = \operatorname{arcsec} x \ \Rightarrow\ \dfrac{dy}{dx} = \dfrac{1}{x\sqrt{x^2-1}}$

$y = \operatorname{arccsc} u \ \Rightarrow\ \dfrac{dy}{dx} = -\dfrac{1}{u\sqrt{u^2-1}}\dfrac{du}{dx}$ $\qquad y = \operatorname{arccsc} x \ \Rightarrow\ \dfrac{dy}{dx} = -\dfrac{1}{x\sqrt{x^2-1}}$

Derivada de funciones implícitas

En muchos problemas de derivadas, la relación entre las variables x, y *no se resuelve para* y*, por lo tanto la variable* y *se llama función implícita de* x*.*

Por ejemplo, $3x^2 - 7y = 0$ *es una relación implícita de* y*. Al resolver la ecuación, se obtiene* $y = \dfrac{3}{7}x^2$ *donde y es función explícita de* x*. Ahora se procede a derivar esta última función, si tal fuere el caso.*

Pero cuando no se puede resolver la ecuación para "y" en forma explícita, se deriva la ecuación dada término a término, considerando a "y" como función de x*. En la ecuación resultante se despeja* $\dfrac{dy}{dx}$*.*

Por ejemplo, si $x^2 + xy = xy^3$ *no es posible expresar* y *como función explícita de* x*.*

Así que $x^2 + xy = xy^3 \ \Rightarrow\ 2x + y + x\dfrac{dy}{dx} = y^3 + 3xy^2\dfrac{dy}{dx}$

$$2x + y - y^3 = 3xy^2\frac{dy}{dx} - x\frac{dy}{dx} \quad \Rightarrow \quad 2x + y - y^3 = \frac{dy}{dx}(3xy^2 - x) \quad \Rightarrow \quad \frac{dy}{dx} = \frac{2x + y - y^3}{3xy^2 - x}$$

$Regla\ de\ la\ cadena$

$Si\ y = f(u)\ \ y\ \ u = h(x),\ \ se\ dice\ que\ \ y\ es\ función\ de\ \ x\ \ por\ intermedio\ de\ u.$

$Ahora,\ \ si\ \ en\ \ y = f(u)\ \ se\ sustituye\ \ u\ \ por\ \ h(x)\ \ se\ tiene\ \ y = f\big(h(x)\big)\ \ que\ se\ llama$

$función\ compuesta\ de\ f\ \ y\ \ h\ \ ,\ \ también\ denominada\ \ función\ de\ función.$

$$Cuando\ \ y = f\big(h(x)\big)\ \ entonces\ \ \frac{dy}{dx} = f'\big(h(x)\big)h'(x)$$

$$si\ \ \ y = f\Big(g\big(h(x)\big)\Big)\ \ entonces\ \ \frac{dy}{dx} = f'\Big(g\big(h(x)\big)\Big)g'\big(h(x)\big)h'(x)$$

$Ejemplo$: $Hallar\ la\ derivada\ de\ y = \operatorname{sen}\big(\sqrt{x}\big).$

$Esta\ función\ puede\ escribirse\ en\ la\ forma\ \ y = f\big(h(x)\big)\ \ donde\ \ f\big(h(x)\big) = \operatorname{sen}\big(h(x)\big)$

$$y\ \ h(x) = \sqrt{x}\ .\ \ Aquí\ \ f'\big(h(x)\big) = \cos\big(h(x)\big)\ \ y\ \ \ h'(x) = \frac{1}{2\sqrt{x}}$$

$$por\ lo\ tanto\ \frac{dy}{dx} = f'\big(h(x)\big)h'(x) \quad \Rightarrow \quad \frac{dy}{dx} = \cos\big(\sqrt{x}\big)\frac{1}{2\sqrt{x}}$$

DERIVAR Y SIMPLIFICAR LAS SIGUIENTES FUNCIONES

$1.-\ Derivar\ y\ simplificar\quad y = \operatorname{sen}\left(3x + \frac{\pi}{2}\right).$

$Solución:\ \ y = \operatorname{sen} u \ \Rightarrow\ \frac{dy}{dx} = \cos u \frac{du}{dx},\ \ aquí\ \ u = 3x + \frac{\pi}{2}\ \ y\ \ u' = 3$

$$entonces\quad y = \operatorname{sen}\left(3x + \frac{\pi}{2}\right)\quad \Rightarrow\quad y' = \cos\left(3x + \frac{\pi}{2}\right)(3)$$

$$y' = 3\cos(3x + \frac{\pi}{2})$$

$2.-Derivar\ y\ simplificar\quad y = (3x^2 - 1)^2$

$Solución:\ Si\ se\ hace\ el\ camio\ de\ variable\ \ u = 3x^2 - 1\ \ y\ \ \ u' = 6x$

$$se\ tendrá\quad y = u^2\ \ y\ \ por\ tanto\quad y' = 2uu',$$

$$entonces\ \ y = (3x^2 - 1)^2\quad \Rightarrow\ \ y' = 2(3x^2 - 1)6x = 12x(3x^2 - 1)$$

$3.-\ Derivar\ y\ simplificar\quad y = x^2.e^x$

$Solución:\ Derivada\ de\ un\ producto:\ y = u \cdot v\ \ \Rightarrow\ \ y' = u'v + uv'$

$Si\ \ u = x^2\ \ y\ \ v = e^x,\ \ entonces\ \ y = x^2e^x\ \ \rightarrow\ \ y' = 2xe^x + x^2.e^x\ \ \rightarrow\ \ y' = xe^x(2 + x)$

$4.-Derivar\ y\ simplificar\quad y = 4x^3 - 3x$

$Solución:\quad Derivada\ de\ una\ suma:\ y = u + u\ \ \Rightarrow\ \ \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$$aquí\ \ u' = 12x\ \ y\ \ v' = -3,\ \ luego\ \ y = 4x^3 - 3x\ \ \Rightarrow\ \ y' = 12x^2 - 3 = 3(4x^2 - 1)$$

$5.-Derivar\ y\ simplificar\quad y = \ln(x + e^x)$

$Solución:\ \ y = \ln u\ \ \Rightarrow\ \ \frac{dy}{dx} = \frac{1}{u}\frac{du}{dx},\qquad como\ \ u = x + e^x\ ,\ \ u' = 1 + e^x$

$$como\ \ y = \ln(u)\ \Rightarrow\ y' = \frac{1}{u} \cdot u'\ ,\ \ entonces\ \ y = \ln(x + e^x)\quad \Rightarrow\ \ y' = \frac{1}{x + e^x} \cdot (1 + e^x)$$

$6.-Derivar\ y\ simplificar\quad y=\ln(x^2.e^x)$

$Solución:\ y=\ln u \ \Rightarrow\ \dfrac{dy}{dx}=\dfrac{1}{u}\dfrac{du}{dx},\qquad donde\ \ u=x^2e^x\ \ y\ \ u'=2xe^x+x^2e^x$

$$así\ que\ y=\ln u\Rightarrow\ y'=\frac{1}{u}\cdot u',\quad luego\ \ y=\ln(x^2e^x)\ \ \Rightarrow\quad y'=\frac{1}{x^2e^x}\cdot(2xe^x+x^2e^x)$$

$$y'=\frac{xe^x(2+x)}{x^2e^x}\ \ \to\ \ y'=\frac{2+x}{x}$$

$7.-Derivar\ y\ simplificar\ \ y=\sqrt[3]{1-6x^5}$

$Solución:\ y=\sqrt[3]{1-6x^5}\ \ \Rightarrow\ \ y=(1-6x^5)^{\frac{1}{3}}\ ,\ \ donde\ \ u=1-6x^5\ \ \ y\ \ u'=30x^4$

$$luego\ y=(1-6x^5)^{\frac{1}{3}}\ \ \Rightarrow\ \ y=u^{\frac{1}{3}}\ \ \Rightarrow\ \ y'=\frac{1}{3}u^{-\frac{2}{3}}\cdot u'\ \ \ \Rightarrow\ \ y'=\frac{1}{3}(1-6x^5)^{-\frac{2}{3}}(30x^4)$$

$$y'=\frac{30x^4}{3\sqrt[3]{(1-6x^5)^2}}\qquad\Rightarrow\qquad y'=\frac{10x^4}{\sqrt[3]{(1-6x^5)^2}}$$

$8.-Derivar\ y\ simplificar\quad y=\dfrac{a}{x}+\dfrac{x}{a}$

$Solución:\ \ y=\dfrac{a}{x}+\dfrac{x}{a}\quad\Rightarrow\quad y=ax^{-1}+\dfrac{x}{a}\quad\Rightarrow\quad y'=-ax^{-2}+\dfrac{1}{a}$

$$y'=-\frac{a}{x^2}+\frac{1}{a}\quad\Rightarrow\ \ y'=\frac{-a^2+x^2}{ax^2}=\frac{x^2-a^2}{ax^2}$$

$9.-Derivar\ y\ simplificar\quad 2x^2-3xy+4y^2=\sqrt{3}$

$Solución:\ \ 2x^2-3xy+4y^2=\sqrt{3}\ \ \Rightarrow\ \ 4x-3(y)-3xy'+8yy'=0$

$$8yy'-3xy'=3y-4x\ \ \Rightarrow\ \ y'(8y-3x)=3y-4x\ \ \Rightarrow\ \ y'=\frac{3y-4x}{8y-3x}$$

$10.-Derivar\ y\ simplificar\quad x^4=\dfrac{x-y}{x+y}$

$Solución:\ \ x^4=\dfrac{x-y}{x+y}\ \ \to\ \ x^4(x+y)=x-y$

$$4x^3(x+y)+x^4(1+y')=1-y'$$

$$4x^3(x+y)+x^4+x^4y'=1-y'$$

$$\rightarrow\ x^4y'+y'=1-4x^3(x+y)-x^4 \quad \rightarrow\ y'=\frac{1-4x^3(x+y)-x^4}{x^4+1}$$

$11.-Derivar\ y\ simplificar\quad x^2y-xy=1-3x^2$

$Solución$: $x^2y-xy=1-3x^2\ \rightarrow\ 2xy+x^2y'-y-xy'=-6x$

$$x^2y'-xy'=-6x-2xy+y$$

$$y'(x^2-x)=-(6x+2xy-y)\ \Rightarrow\ y'=-\frac{(6x+2xy-y)}{x(x-1)}$$

$12.-Derivar\ y\ simplificar\quad y=\sqrt{\operatorname{sen} x}$

$Solución$: $y=\sqrt{\operatorname{sen} x}\ \rightarrow\ y'=\dfrac{\cos x}{2\sqrt{\operatorname{sen} x}}$

$$y'=\frac{\sqrt{\operatorname{sen} x}\cos x}{2\operatorname{sen} x}\ \rightarrow\ y'=\frac{1}{2}\sqrt{\operatorname{sen} x}\cot x$$

$13.-Derivar\ y\ simplificar\quad y=2\operatorname{sen}(3x)\cos(3x)$

$Solución: recordando\ que\quad \operatorname{sen}(2x)=2\operatorname{sen} x\cos x$

$$y=2\operatorname{sen}(3x)\cos(3x)\quad\Rightarrow\quad y=\operatorname{sen}(6x)\ \Rightarrow\ y'=6\cos(6x)$$

$14.-Derivar\ y\ simplificar\ \ x^2\cos^3(4x)=x^3-y^3$

$Solución$:

$$x^2\cos^3(4x)=x^3-y^3\ \Rightarrow\ (2x)\cos^3(4x)+3x^2[3\cos^2(4x)(-4\operatorname{sen}(4x))]=3x^2-3y^2y'$$

$$2x\cos^3(4x)-12x^2\cos^2(4x)\operatorname{sen}(4x)-3x^2+3y^2y'=0$$

$$y'=\frac{12x^2\cos^2(4x)\operatorname{sen}(4x)-2x\cos^3(4x)+3x^2}{3y^2}$$

15. −Derivar y simplificar $y = \operatorname{sen}\left(\cos\sqrt{x^3}\right)$

Solución: $y = \operatorname{sen}\left(\cos\sqrt{x^3}\right) \Rightarrow y' = \cos\left(\cos\sqrt{x^3}\right)\left(-\operatorname{sen}\sqrt{x^3}\right)\dfrac{3x^2}{2\sqrt{x^3}}$

$$y' = -\frac{3x^2\cos(\cos\sqrt{x^3})\operatorname{sen}\sqrt{x^3}}{2\sqrt{x^3}} \Rightarrow y' = -\frac{3x^2\cos(\cos\sqrt{x^3})\operatorname{sen}\sqrt{x^3}}{2x\sqrt{x}}$$

$$y' = -\frac{3x\cos(\cos\sqrt{x^3})\operatorname{sen}\sqrt{x^3}}{2\sqrt{x}}$$

16. $-Derivar\ y\ simplificar$ $y = \sec^2 x\tan^2 x$

Solución: *recordando que* $\tan^2 x = \sec^2 x - 1$

$$y = \sec^2 x\tan^2 x \Rightarrow y = \sec^2 x[\sec^2 x - 1] \Rightarrow y = \sec^4 x - \sec^2 x$$

$$y' = 4\sec^3 x\sec x\tan x - 2\sec x\,.\sec x\tan x \Rightarrow y' = 2\sec^2 x\tan x[2\sec^2 x - 1]$$

17. $-Derivar\ y\ simplificar$ $y = \dfrac{x^2+1}{\sqrt{1-x^2}}$

Solución: $y = \dfrac{x^2+1}{\sqrt{1-x^2}} \rightarrow y' = \dfrac{\sqrt{1-x^2}(2x) - (x^2+1).\frac{(-2x)}{2\sqrt{1-x^2}}}{1-x^2}$

$$y' = \frac{2x\sqrt{1-x^2} + \frac{x(x^2+1)}{\sqrt{1-x^2}}}{1-x^2} \Rightarrow y' = \frac{2x(1-x^2) + x(x^2+1)}{(1-x^2)\sqrt{1-x^2}}$$

$$y' = \frac{2x - 2x^3 + x^3 + x}{(1-x^2)\sqrt{1-x^2}} = \frac{3x - x^3}{(1-x^2)\sqrt{1-x^2}}$$

18. $-Derivar\ y\ simplificar$ $\sec(xy) = xy$

Solución: $\sec(xy) = xy \rightarrow \sec(xy)\tan(xy)[y + y'x] = y + xy'$

$$y'(x\sec(xy)\tan(xy) - x) = y - y\sec(xy)\tan(xy)$$

$$y\sec(xy)\tan(xy) + y'x\sec(xy)\tan(xy) - y - xy' = 0$$

$$y' = \frac{y(1 - \sec(xy)\tan(xy))}{x(\sec(xy)\tan(xy) - 1)} = -\frac{y}{x}.\frac{\sec(xy)\tan(xy) - 1}{\sec(xy)\tan(xy) - 1}$$

19. $-$*Derivar y simplificar* $y = (2x^3-1)^2\sqrt[4]{x^2-3}$

Solución:

$$y = (2x^3-1)^2\sqrt[4]{x^2-3} \;\rightarrow\; y' = 2(2x^3-1)(6x^2)\sqrt[4]{x^2-3} + \frac{(2x^3-1)^2(2x)}{4\sqrt[4]{(x^2-3)^3}}$$

$$y' = x(2x^3-1)\left[\frac{24x(x^2-3)+2x^3-1}{2\sqrt[4]{x^2-3}}\right]$$

$$y' = x(2x^3-1)\left[\frac{24x^3-72x+2x^3-1}{2\sqrt[4]{(x^2-3)^3}}\right] \;\Rightarrow\; y' = x(2x^3-1)\left[\frac{26x^3-72x-1}{2\sqrt[4]{(x^2-3)^3}}\right]$$

20. $-$*Derivar y simplificar* $y^2\cos x = a^2\operatorname{sen}(bx)$

Solución: $y^2\cos x = a^2\operatorname{sen}(bx) \;\rightarrow\; 2yy'\cos x - y^2\operatorname{sen} x = a^2\cos(bx)b$

$$\rightarrow\; 2yy'\cos x = ba^2\cos(bx) + y^2\operatorname{sen} x$$

$$\rightarrow\; y' = \frac{a^2b\cos(bx)+y^2\operatorname{sen} x}{2y\cos x}$$

DERIVADAS DE FUNCIONES TRASCENDENTES

$$Derivar\ y\ simplificar\ las\ funciones\ que$$
$$se\ muestran\ a\ continuación.$$

$1.-Derivar\ y\ simplificar\ \ y = \operatorname{sen}\,(7x - \frac{\pi}{2})$

$Solución$: $y = \operatorname{sen}\left(7x - \frac{\pi}{2}\right) \;\Rightarrow\; y' = \cos\left(7x - \frac{\pi}{2}\right)\frac{d}{dx}\,(7x - \frac{\pi}{2})$

$$y' = \cos\left(7x - \frac{\pi}{2}\right)\,(7) \quad \Rightarrow \quad y' = 7\cos\,\left(7x - \frac{\pi}{2}\right)$$

$2.-Derivar\ y\ simplificar\ \ y = x^2.e^{x^2}$

$Solución$: $y = x^2.e^{x^2} \;\Rightarrow\; y' = \frac{d}{dx}(x^2).e^{x^2} + x^2.\frac{d}{dx}\left(e^{x^2}\right) \;\Rightarrow\; y' = 2x.e^{x^2} + x^2.e^{x^2}(2x)$

$$y' = 2x.e^{x^2} + 2x^3.e^{x^2} \quad \Rightarrow \quad y' = 2xe^{x^2}(1 + x^2)$$

$3.-Derivar\ y\ simplificar\ \ y = \operatorname{arcsec}(3x^2 + 2)^5$

$Solución$: $y = \operatorname{arcsec}(3x^2 + 2)^5 \;\Rightarrow\; y' = \frac{d(3x^2+2)^5}{(3x^2+2)^5\sqrt{(3x^2+2)^{10} - 1}}$

$$y' = \frac{5(3x^2+2)^4\,(6x)}{(3x^2+2)^5\sqrt{(3x^2+2)^{10} - 1}} \quad \Rightarrow \quad y' = \frac{30x}{(3x^2+2)\sqrt{(3x^2+2)^{10} - 1}}$$

Nota: recordemos que si $y = \sec^{-1}(u)$, $entonces\ y' = \frac{u'}{u\sqrt{u^2 - 1}}$

$4.-Derivar\ y\ simplificar\ \ y = \log(\sqrt{x} - 1)$

$Solución$: $Si\ \ u = \sqrt{x} - 1\,,\quad entonces\ \ \frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$$por\ tanto\ \ y = \log(u)\ \Rightarrow y' = \frac{\log\,(e)}{u}\cdot\frac{du}{dx} \;\Rightarrow\; y' = \frac{1}{(\sqrt{x} - 1)}\cdot\frac{\log\,(e)}{2\sqrt{x}}\,.$$

$$y' = \frac{1}{2}\cdot\frac{\log\,(e)}{(x - \sqrt{x})} \quad \Rightarrow \quad y' = \frac{\log\,(\mathrm{e})}{2}\cdot\frac{1}{(x - \sqrt{x})}$$

$5. -Derivar\ y\ simplificar\ \ y = \text{arcsen}(e^{x^2})$

$Solución: Si\ \ u = x^2 \quad entonces \quad \dfrac{du}{dx} = 2x$

$$luego\ \ y = \text{arcsen}(e^u) \Rightarrow y' = \frac{e^u}{\sqrt{1-(e^u)^2}} \cdot \frac{du}{dx} \ \Rightarrow\ y' = \frac{(e^{x^2})(2x)}{\sqrt{1-\left(e^{x^2}\right)^2}} \ \Rightarrow\ y' = \frac{2xe^{x^2}}{\sqrt{1-e^{2x^2}}}$$

$6. -Derivar\ y\ simplificar\ \ y = \text{sen}\ (\text{arccos}(x+1))$

$Solución$:

$$y = \text{sen}(\text{arccos}(x+1)) \ \Rightarrow\ y' = \cos(\text{arccos}(x+1))\,.\frac{-1}{\sqrt{1-(x+1)^2}}$$

$$y' = \frac{-(x+1)}{\sqrt{1-x^2-2x-1}} \Rightarrow \quad y' = \frac{-(x+1)}{\sqrt{-x^2-2x}} = -\frac{(x+1)}{\sqrt{-x^2-2x}}$$

$7. -Derivar\ y\ simplificar\ \ y = e^{\sqrt{x}}$

$Solución:\ Si\ \ u = \sqrt{x}\ \ entonces \quad \dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}}$

$$así\ que\ \ \ y = e^u \quad \Rightarrow \quad y' = e^u \cdot \frac{du}{dx} \ \Rightarrow\ y' = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} \ \Rightarrow\ y' = \frac{1}{2} \cdot \frac{e^{\sqrt{x}}}{\sqrt{x}}$$

$8. -$ Derivar y simplificar $\quad y = \ln(x + e^x)$

$Solución:\ y = \ln(x+e^x) \ \Rightarrow\ y' = \dfrac{1}{x+e^x} \cdot (1+e^x)$

$$y' = \frac{1+e^x}{x+e^x}$$

$9. -$ Derivar y simplificar $\arctan\left(\dfrac{x}{y}\right) + 2x^2y = \dfrac{y}{x}$

Solución: $\arctan\left(\dfrac{x}{y}\right) + 2x^2y = \dfrac{y}{x} \ \rightarrow\ \dfrac{1}{\frac{y^2+x^2}{y^2}} \cdot \dfrac{y-xy'}{y^2} + 4xy + 2x^2y' = \dfrac{xy'-y}{x^2}$

$$\frac{y - xy'}{y^2 + x^2} + 4xy + 2x^2y' = \frac{xy' - y}{x^2}$$

$$x^2(y - xy') + (4xy + 2x^2y')(x^2)(x^2 + y^2) = (x^2 + y^2)(xy' - y)$$

$$x^2y - x^3y' + 4xy(x^2)(x^2 + y^2) + 2x^2y'(x^2)(x^2 + y^2) = (x^2 + y^2)xy' - y(x^2 + y^2) \rightarrow$$

$$2x^2y'(x^2)(x^2 + y^2) - x^3y' - (x^2 + y^2)xy' = -[y(x^2 + y^2) + 4xy(x^2)(x^2 + y^2) + x^2y] \rightarrow$$

$$y' = -\frac{[(y + 4x^3y)(x^2 + y^2) + x^2y]}{[(2x^4 - x)(x^2+y^2) - x^3]} = -\frac{y[(1 + 4x^3)(x^2 + y^2) + x^2]}{x[(2x^3 - 1)(x^2 + y^2) - x^2]}$$

10. $-$*Derivar y simplificar* $y = \ln(\ln(\sqrt{x}))$

Solución I: $y = \ln(\ln(\sqrt{x})) \Rightarrow y' = \frac{1}{\ln\sqrt{x}} \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$

$$y' = \frac{1}{2x \ln\sqrt{x}} \quad \Rightarrow \quad y' = \frac{1}{x \ln x}$$

Solución II: $y = \ln(\ln(\sqrt{x})) \quad \Rightarrow \quad y = \ln\left(\frac{1}{2}\ln(x)\right)$

$$y' = \frac{1}{0.5 \ln x} \cdot \ 0.5\frac{1}{x}$$

$$y' = \frac{1}{x \ln x}$$

11. $-$*Derivar y simplificar* $\operatorname{arcsen}(x.y) = \arccos(x + y)$

Solución: $\operatorname{arcsen}(x.y) = \arccos(x + y) \Rightarrow \frac{y + xy'}{\sqrt{1 - (xy)^2}} = \frac{(-1)(1 + y')}{\sqrt{1 - (x + y)^2}}$

$$\frac{y}{\sqrt{1 - (xy)^2}} + \frac{xy'}{\sqrt{1 - (xy)^2}} + \frac{1}{\sqrt{1 - (x + y)^2}} + \frac{y'}{\sqrt{1 - (x + y)^2}} = 0$$

$$y'\left(\frac{x}{\sqrt{1 - (xy)^2}} + \frac{1}{\sqrt{1 - (x + y^2}}\right) = -\frac{y}{\sqrt{1 - (xy)^2}} - \frac{1}{\sqrt{1 - (x + y)^2}}$$

$$y'\left(\frac{x.\sqrt{1 - (x + y^2} + \sqrt{1 - (xy)^2}}{\sqrt{1 - (xy)^2}\sqrt{1 - (x + y^2}}\right) = -\frac{y\sqrt{1 - (x + y)^2} + \sqrt{1 - (xy)^2}}{\sqrt{1 - (xy)^2}.\sqrt{1 - (x + y)^2}}$$

$$y' = -\frac{y\sqrt{1-(x+y)^2}+\sqrt{1-(xy)^2}}{x.\sqrt{1-(x+y^2}+\sqrt{1-(xy)^2}}$$

12.$-$*Derivar y simplificar* $\operatorname{arcsen}(\sqrt{x}) = \operatorname{arcsec}(\sqrt{y})$

Solución: $\operatorname{arcsen}(\sqrt{x}) = \operatorname{arcsec}(\sqrt{y}) \;\to\; \frac{1}{\sqrt{1-x}}\cdot\frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{y}\sqrt{y-1}}\cdot\frac{y'}{2\sqrt{y}}$

$$\frac{1}{2\sqrt{x}\sqrt{1-x}} = \frac{y'}{2y\sqrt{y-1}}$$

$$y' = \frac{y\sqrt{y-1}}{\sqrt{x-x^2}} \;\to\; y' = y\sqrt{\frac{y-1}{x-x^2}}$$

13.$-$*Derivar y simplificar* $y = \arctan\sqrt{\frac{1-\cos x}{1+\cos x}}$

Solución: $y = \arctan\sqrt{\frac{1-\cos x}{1+\cos x}}$

$$y' = \frac{1}{1+\frac{1-\cos x}{1+\cos x}}\cdot\frac{1}{2\sqrt{\frac{1-\cos x}{1+\cos x}}}\cdot\frac{\operatorname{sen} x(1+\cos x)-(-\operatorname{sen} x)(1-\cos x)}{(1+\cos x)^2}$$

$$y' = \frac{1}{\frac{1+\cos x+1-\cos x}{1+\cos x}}\cdot\frac{1}{2\sqrt{\frac{1-\cos x}{1+\cos x}}}\cdot\frac{\operatorname{sen} x+\operatorname{sen} x\cos x+\operatorname{sen} x-\operatorname{sen} x\cos x}{(1+\cos x)^2}$$

$$y' = \frac{1}{\frac{2}{1+\cos x}}\cdot\frac{1}{2\sqrt{\frac{(1-\cos x)^2}{1-\cos^2 x}}}\cdot\frac{2\operatorname{sen} x}{(1+\cos x)^2}$$

$$y' = \frac{1+\cos x}{2}\cdot\frac{1}{2\frac{(1-\cos x)}{\operatorname{sen} x}}\cdot\frac{2\operatorname{sen} x}{(1+\cos x)^2} = \frac{1}{2}\cdot\frac{1}{1-\cos x}\cdot\frac{\operatorname{sen}^2 x}{1+\cos x} = \frac{1}{2}$$

$$y = \arctan\sqrt{\frac{1-\cos x}{1+\cos x}} \quad\Rightarrow\quad y' = \frac{1}{2}$$

14. − Derivar y simplificar $y = \sqrt{x^2-1} + \operatorname{arcsec}(\sqrt{x^2-1})$

Solución:

$$y = \sqrt{x^2-1} + \operatorname{arcsec}(\sqrt{x^2-1}) \;\Rightarrow\; y' = \frac{2x}{2\sqrt{x^2-1}} + \frac{1}{\sqrt{x^2-1}\sqrt{(x^2-1)-1}} \cdot \frac{2x}{2\sqrt{x^2-1}}$$

$$y' = \frac{x}{\sqrt{x^2-1}} + \frac{x}{(x^2-1)\sqrt{x^2-2}} = \frac{x\sqrt{x^2-1}}{x^2-1} + \frac{x}{(x^2-1)\sqrt{x^2-2}}$$

$$y' = \frac{x}{x^2-1}\left[\sqrt{x^2-1} + \frac{1}{\sqrt{x^2-2}}\right]$$

15. −*Derivar y simplificar* $y = \dfrac{(\ln(x))^2}{\ln x}$

Solución: $y = \dfrac{(\ln(x))^2}{\ln(x)} \;\Rightarrow\; y = \dfrac{\ln(x).\ln(x)}{\ln(x)}$

$$y = \ln(x) \;\Rightarrow\; y' = \frac{1}{x}$$

16. −Derivar y simplificar $y = \dfrac{x}{2}\sqrt{a^2-x^2} + \dfrac{a^2}{2}\operatorname{arcsen}\left(\dfrac{x}{a}\right)$

Solución:

$$y = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\operatorname{arcsen}\left(\frac{x}{a}\right) \;\Rightarrow\; y' = \frac{1}{2}\sqrt{a^2-x^2} + \frac{(x)(-2x)}{2.2\sqrt{a^2-x^2}} + \frac{a^2}{2} \cdot \frac{1\left(\frac{1}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}$$

$$y' = \frac{1}{2}\sqrt{a^2-x^2} - \frac{x^2}{2\sqrt{a^2-x^2}} + \frac{a}{2} \cdot \frac{1}{\sqrt{\frac{a^2-x^2}{a^2}}}$$

$$y' = \frac{1}{2}\sqrt{a^2-x^2} - \frac{x^2}{2\sqrt{a^2-x^2}} + \frac{a^2}{2} \cdot \frac{1}{\sqrt{a^2-x^2}}$$

$$y' = \frac{(a^2-x^2) - x^2 + a^2}{2\sqrt{a^2-x^2}} = \frac{a^2-x^2-x^2+a^2}{2\sqrt{a^2-x^2}} = \frac{2(a^2-x^2)}{2\sqrt{a^2-x^2}}$$

$$y' = \frac{(a^2 - x^2)\sqrt{a^2 - x^2}}{(a^2 - x^2)} = \sqrt{a^2 - x^2}$$

17. −Derivar y simplificar $y = \ln(x^2.e^x)$

Solución 1 :

$$y = \ln(x^2.e^x) \;\; \Rightarrow \;\; y' = \frac{1}{x^2.e^x} \cdot (2xe^x + x^2e^x)$$

$$y' = \frac{xe^x(2+x)}{x^2.e^x} \;\; \Rightarrow \;\; y' = \frac{2+x}{x}$$

Solución 2:

$$y = \ln(x^2.e^x) \;\; \Rightarrow \;\; y = \ln x^2 + \ln e^x = 2\ln x + x$$

$$y' = \frac{2}{x} + 1 \;\; \Rightarrow \;\; y' = \frac{2+x}{x}$$

18. −*Derivar y simplificar* $y = \ln(\arctan(e^{3x}))$

Solución:

$$y = \ln(\arctan(e^{3x})) \;\; \rightarrow \;\; y' = \frac{1}{\arctan(e^{3x})} \cdot \frac{1}{1+e^{6x}} \cdot e^{3x}.(3)$$

$$y' = \frac{3e^{3x}}{(1+e^{6x})\arctan(e^{3x})}$$

19. −*Derivar y simplificar* $y = \ln\sqrt[5]{\dfrac{1+\operatorname{sen} x}{1-\operatorname{sen} x}}$

Solución: $y = \ln\sqrt[5]{\dfrac{1+\operatorname{sen} x}{1-\operatorname{sen} x}} = \dfrac{1}{5}\ln\left(\dfrac{1+\operatorname{sen} x}{1-\operatorname{sen} x}\right)$

$$y' = \frac{1}{5} \cdot \frac{1}{\frac{1+\operatorname{sen} x}{1-\operatorname{sen} x}} \cdot \frac{(1-\operatorname{sen} x)\cos x - (1+\operatorname{sen} x)(-\cos x)}{(1-\operatorname{sen} x)^2}$$

$$y' = \frac{1}{5} \cdot \frac{(1-\operatorname{sen} x)}{(1+\operatorname{sen} x)} \cdot \frac{\cos x - \operatorname{sen} x\cos x + \cos x + \operatorname{sen} x\cos x}{(1-\operatorname{sen} x)^2}$$

$$y' = \frac{1}{5}\frac{2\cos x}{(1+\operatorname{sen} x)(1-\operatorname{sen} x)} = \frac{2\cos x}{5\cos^2 x} = \frac{2}{5\cos x}$$

20. − Derivar y simplificar $\ln(x+y)^3 = x^2+y^2$

Solución: $\ln(x+y)^3 = x^2+y^2 \quad \Rightarrow \quad 3\ln(x+y) = x^2+y^2$

$$3\cdot\frac{1}{x+y}\cdot(1+y') = 2x+2yy' \quad \Rightarrow \quad 3(1+y') = 2x(x+y)+2yy'(x+y)$$

$$3y'-2yy'(x+y) = 2x(x+y)-3 \quad \Rightarrow \quad y'\big(3-2y(x+y)\big) = 2x(x+y)-3$$

$$y' = \frac{2x(x+y)-3}{3-2y(x+y)} = -\frac{2x(x+y)-3}{2y(x+y)-3}$$

21. − Derivar y simplificar $\ln(x+y) = e^{xy}+x^2$

Solución; $\ln(x+y) = e^{xy}+x^2 \;\Rightarrow\; \dfrac{1+y'}{x+y} = (y+xy')e^{xy}+2x$

$$\frac{1+y'}{x+y} = ye^{xy}+xy'e^{xy}+2x$$

$$1+y' = (x+y)ye^{xy}+(x+y)xy'e^{xy}+2x(x+y)$$

$$y'-xy'e^{xy}(x+y) = (x+y)ye^{xy}+2x(x+y)-1$$

$$y' = \frac{y(x+y)e^{xy}+2x(x+y)-1}{1-xe^{xy}(x+y)}$$

22. −*Derivar y simplificar* $y = \dfrac{e^x-e^{-x}}{e^x+e^{-x}}$

Solución: $y = \dfrac{e^x-e^{-x}}{e^x+e^{-x}} \;\Rightarrow\; y' = \dfrac{(e^x+e^{-x})(e^x+e^{-x})-(e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2}$

$$y' = \frac{(e^{2x}+2+e^{-2x})-(e^{2x}-2+e^{-2x})}{(e^x+e^{-x})^2}$$

$$\frac{e^{2x}+2+e^{-2x}-e^{2x}+2-e^{-2x}}{(e^x+e^{-x})^2} = \frac{4}{(e^x+e^{-x})^2}$$

23. − Derivar y simplificar $\ln(x^2+y^2) = 2\arctan\left(\dfrac{x}{y}\right)$

Solución: $\ln(x^2+y^2) = 2\arctan\left(\dfrac{x}{y}\right) \;\Rightarrow\; \dfrac{2x+2yy'}{x^2+y^2} = 2\,.\dfrac{1}{1+\frac{x^2}{y^2}}\,.\dfrac{y-xy'}{y^2}$

$$\frac{2x+2yy'}{x^2+y^2}=\frac{2(y-xy')}{x^2+y^2} \quad \Rightarrow \quad \frac{2x+2yy'}{x^2+y^2}=\frac{2y^2}{x^2+y^2}\cdot\frac{y-xy'}{y^2}$$

$$2x+2yy'=2y-2xy' \;\Rightarrow\; 2yy'+2xy'=2y-2x$$

$$2y'(y+x)=2(y-x) \quad \Rightarrow \quad y'=\frac{y-x}{y+x}$$

$24.-Derivar\ y\ simplificar \quad \ln(y)=\operatorname{sen}x$

$Solución$:

$$\ln(y)=\operatorname{sen}x \Rightarrow \frac{y'}{y}=\cos x \quad \Rightarrow y'=y\cos x$$

$25.-Derivar\ y\ simplificar \quad y=\operatorname{arcsen}(\ln x)-\ln(\arctan x)$

$Solución$:

$$y=\operatorname{arcsen}(\ln x)-\ln(\arctan x) \quad \Rightarrow \quad y'=\frac{1}{\sqrt{1-\ln^2 x}}\cdot\frac{1}{x}-\frac{1}{\arctan x}\cdot\frac{1}{1+x^2}$$

$$y'=\frac{1}{\sqrt{1-\ln^2 x}}\cdot\frac{1}{x}-\frac{1}{\arctan x}\cdot\frac{1}{1+x^2}$$

$$y'=\frac{1}{x\sqrt{1-\ln^2 x}}-\frac{1}{(1+x^2)\arctan(x)}$$

$26.-$ Derivar y simplificar$\sqrt{1+\ln y}=\operatorname{sen}^2(3x-1)$

$Solución$:

$$\sqrt{1+\ln y}=\operatorname{sen}^2(3x-1) \;\Rightarrow\; \frac{1}{2\sqrt{1+\ln y}}\cdot\frac{y'}{y}=2\operatorname{sen}(3x-1)\cos(3x-1).3$$

$$y'=12y\operatorname{sen}(3x-1)\cos(3x-1)\sqrt{1+\ln y}$$

$27.-$ Derivar y simplificar $y=\ln[(2x+1)^3.(x^2-4)^5]$

$Solución$:

$$y=\ln[(2x+1)^3\cdot(x^2-4)^5] \quad \Rightarrow \quad y=3\ln(2x+1)+5\ln(x^2-4)$$

$$y' = 3 \cdot \frac{2}{2x+1} + 5 \cdot \frac{2x}{x^2-4} \quad \Rightarrow \quad y' = \frac{6}{2x+1} + \frac{10x}{x^2-4}$$

$$y' = \frac{6x^2 - 24 + 20x^2 + 10x}{(2x+1)(x^2-4)} = \frac{26x^2 + 10x - 24}{(2x+1)(x^2-4)}$$

28. – Derivar y simplificar $y = \text{sen}^3(e^x).\cos^4(e^{-x})$

Solución:

$$y = \text{sen}^3(e^x).\cos^4(e^{-x})$$

$$y' = 3\text{sen}^2(e^x)\cos(e^x)\,e^x.\cos^4(e^{-x}) + 4\cos^3(e^{-x})(-\text{sen}(e^{-x})(e^{-x})(-1)\text{sen}^3(e^x))$$

$$y' = 3\text{sen}^2(e^x)\cos^3(e^{-x})[3\cos(e^x)\cos(e^{-x})\,e^x + 4\,\text{sen}(e^{-x})\,\text{sen}(e^x)e^{-x}]$$

$$y' = \text{sen}^2(e^x)\cos^3(e^{-x})[3e^x\cos(e^x)\cos(e^{-x}) + 4e^{-x}\,\text{sen}(e^{-x})\,\text{sen}(e^x)]$$

29. – Derivar y simplificar $\ln\sqrt{x.y} + \ln\sqrt{x+y} = 4$

Solución: $\ln\sqrt{x.y} + \ln\sqrt{x+y} = 4 \;\Rightarrow\; \frac{1}{2}\ln(x.y) + \frac{1}{2}\ln(x+y) = 4$

$$\ln(x.y) + \ln(x+y) = 8 \quad \Rightarrow \quad \frac{y+xy'}{x.y} + \frac{1+y'}{x+y} = 0$$

$$(y+xy')(x+y) + (1+y')(x.y) = 0 \quad \Rightarrow \quad y(x+y) + xy'(x+y) + xy + xyy' = 0$$

$$y'[x(x+y) + xy] = -y(x+y) - xy$$

$$y' = -\frac{y(x+y)+xy}{x(x+y)+xy} \quad \Rightarrow \quad y' = -\frac{2xy+y^2}{2xy+x^2}$$

30. – Derivar y simplificar $\text{arcsen}[\ln(x+y)] = x \cdot y$

Solución: $\text{arcsen}[\ln(x+y)] = xy \;\Rightarrow\; \frac{1}{\sqrt{1-\ln^2(x+y)}} \cdot \frac{1}{x+y} \cdot (1+y') = y + xy'$

$$1 + y' = (y+xy')(x+y)\sqrt{1-\ln^2(x+y)}$$

$$1 + y' = y(x+y)\sqrt{1-\ln^2(x+y)} + xy'(x+y)\sqrt{1-\ln^2(x+y)}$$

$$y' - xy'(x+y)\sqrt{1-\ln^2(x+y)} = y(x+y)\sqrt{1-\ln^2(x+y)} - 1$$

$$y'(1 - x(x+y)\sqrt{1 - \ln^2(x+y)} = y(x+y)\sqrt{1 - \ln^2(x+y)} - 1$$

$$y' = \frac{y(x+y)\sqrt{1 - \ln^2(x+y)} - 1}{1 - x(x+y)\sqrt{1 - \ln^2(x+y)}} = -\frac{y(x+y)\sqrt{1 - \ln^2(x+y)} - 1}{x(x+y)\sqrt{1 - \ln^2(x+}y) - 1}$$

31.− Derivar y simplificar $y = 10e^{-x}.\cos x$

Solución:

$$y = 10e^{-x}.\cos x \;\Rightarrow\; y' = -10e^{-x}.\cos x - 10e^{-x}.\operatorname{sen} x$$

$$y' = -10e^{-x}(\cos x + \operatorname{sen} x)$$

FUNCIONES HIPERBOLICAS

Las funciones hiperbólicas se definen como se muestra a continuación:

$$\operatorname{senh}(x) = \frac{e^x - e^{-x}}{2} \qquad \cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \tanh(x) = \frac{\operatorname{senh}(x)}{\cosh(x)}$$

$$\coth(x) = \frac{\cosh(x)}{\operatorname{senh}(x)} \qquad \operatorname{sech}(x) = \frac{1}{\cosh(x)} \qquad \operatorname{csch}(x) = \frac{1}{\operatorname{senh}(x)}$$

$$\cosh^2 x - \operatorname{senh}^2 x = 1 \qquad \cosh(2x) = \cosh^2 x + \operatorname{senh}^2 x \qquad \operatorname{senh}(2x) = 2\operatorname{senh} x . \cosh x$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x \qquad \coth^2 x - 1 = \csc^2 x$$

Derivadas de las funciones hiperbólicas:

$$y = \operatorname{senh}(x) \Rightarrow y' = \cosh(x) \qquad y = \cosh(x) \Rightarrow y' = \operatorname{senh}(x)$$

$$y = \tanh(x) \Rightarrow y' = \operatorname{sech}^2(x) \qquad y = \coth(x) \Rightarrow y' = -\operatorname{csch}^2(x)$$

$$y = \operatorname{sech}(x) \Rightarrow y' = -\operatorname{sech}(x)\tanh(x) \qquad y = \operatorname{csch}(x) \Rightarrow y' = -\operatorname{csch}(x)\coth(x)$$

Derivadas de las funciones hiperbólicas inversas:

$$y = \operatorname{arcsenh}(u) \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{u^2+1}}\frac{du}{dx} \qquad y = \operatorname{arccosh}(u) \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx}$$

$$y = \operatorname{arctanh}(u) \Rightarrow \frac{dy}{dx} = \frac{1}{1-u^2}\frac{du}{dx} \qquad y = \operatorname{arcoth}(u) \Rightarrow \frac{dy}{dx} = \frac{1}{1-u^2}\frac{du}{dx}$$

$$y = \operatorname{arcsech}(u) \Rightarrow \frac{dy}{dx} = \frac{1}{|u|\sqrt{1-u^2}}\frac{du}{dx} \qquad y = \operatorname{arccsch}(u) \Rightarrow \frac{dy}{dx} = \frac{1}{|u|\sqrt{1+u^2}}\frac{du}{dx}$$

EJERCICIOS

1. −Derivar y simplificar $y = x \operatorname{senh} x + e^{\cosh x}$

$$Solución:\ y = x \operatorname{senh}(x) + e^{\cosh x} \quad \Rightarrow \quad y' = \operatorname{senh}(x) + x \cosh(x) + \operatorname{senh}(x)\, e^{\cosh x}$$

$$y' = \operatorname{senh}(x)\left[1 + e^{\cosh(x)}\right] + x.\cosh(x)$$

2. −Derivar y simplificar $y = \operatorname{senh}(\arctan(e^{3x}))$

$Solución:\ si \quad u = \arctan(e^{3x})\,, \quad u' = \dfrac{3e^{3x}}{1 + e^{6x}}$

$$entonces \quad y = \operatorname{senh}(\arctan(e^{3x})) \quad \Rightarrow \quad y = \operatorname{senh}(u)$$

$$así\ que \quad y' = \cosh(u)\ .\ \frac{du}{dx} \Rightarrow \quad y' = \cosh(\arctan(e^{3x}))\,.\frac{3.\,e^{3x}}{1 + e^{6x}}$$

$$y' = \frac{3e^{3x}\cosh(\arctan(e^{3x}))}{1 + e^{6x}}$$

3. −Derivar y simplificar $y = \operatorname{arcsen}(\tanh(x^2))$

$$Solución: y = \operatorname{arcsen}(\tanh(x^2)) \quad \Rightarrow \quad y' = \frac{2.x.\operatorname{sech}^2(x^2)}{\sqrt{1 - \tanh^2(x^2)}} \ \rightarrow$$

$$y' = \frac{2x\operatorname{sech}^2(x^2)}{\sqrt{1 - \tanh^2(x^2)}} = 2x.\operatorname{sech}^2(x^2).\operatorname{sech}^{-1}(x^2) = 2x\operatorname{sech}(x^2)$$

4. $-Derivar\ y\ simplificar\ \ y = \dfrac{\operatorname{senh} x}{1 + \cosh x}$

$$Solución:\ y = \frac{\operatorname{senh} x}{1 + \cosh x} \quad \Rightarrow \quad y' = \frac{[1 + \cosh x]\cosh x - \operatorname{senh} x \operatorname{senh} x}{[1 + \cosh x]^2}$$

$$y' = \frac{\cosh x + \cosh^2 x - \operatorname{senh}^2 x}{[1 + \cosh x]^2} = \frac{\cosh x + 1}{[1 + \cosh x]^2} = \frac{1}{1 + \cosh x}$$

5. −Derivar y simplificar $\operatorname{senh} y = \tan x$

$$Solución:\ \operatorname{senh} y = \tan x \ \Rightarrow \ y' \cosh y = \sec^2(x) \quad \Rightarrow \quad y' = \frac{\sec^2(x)}{\cosh y}$$

6. −Derivar y simplificar $y = \operatorname{arccoth}\left[\ln\left(\frac{x}{2}\right)\right]$

Solución: $y = \operatorname{arccoth}\left[\ln\left(\frac{x}{2}\right)\right] \quad \Rightarrow \quad y' = \dfrac{1}{1-\ln^2\left(\frac{x}{2}\right)} \cdot \dfrac{1}{\frac{x}{2}} \cdot \dfrac{1}{2}$

$$y' = \frac{1}{x\left[1-\ \ln^2\left(\frac{x}{2}\right)\right]}$$

7. −Derivar y simplificar $y = \ln\sqrt{\tanh(2x)}$
Solución: $si\ u = \ \tanh(2x)$, $u' = 2\operatorname{sech}^2(2x)$

luego $\quad y = \ln\sqrt{u} \quad \Rightarrow \quad y' = \dfrac{1}{\sqrt{u}} \cdot \dfrac{1}{2\sqrt{u}}\dfrac{du}{dx}$, *como* $u' = \ 2.\operatorname{sech}^2(2x)$

$y = \ln\sqrt{\tanh(2x)} \ \Rightarrow \ y' = \dfrac{1}{\sqrt{\tanh(2x)}} \cdot \dfrac{1}{2\sqrt{\tanh(2x)}} \cdot \operatorname{sech}^2(2x).2$

$$y' = \frac{\operatorname{sech}^2(2x)}{\tanh(2x)} = \frac{1}{\cosh(2x)\operatorname{senh}(2x)} \quad \Rightarrow \quad y' = \frac{2}{\operatorname{senh}(4x)}$$

8. −Derivar y simplificar $y = \left[\sqrt[3]{\operatorname{sech} x}\right]^4$
Solución: $y = \left[\sqrt[3]{\operatorname{sech} x}\right]^4 \ \rightarrow \ y = (\operatorname{sech} x)^{4/3}$

$$y' = \ \frac{4}{3}(\operatorname{sech} x)^{\frac{1}{3}}.(-\operatorname{sech} x \tanh(x)) \quad \Rightarrow \ y' = -\frac{4}{3}(\operatorname{sech} x)^{\frac{4}{3}}.\tanh(x)$$

9. −Derivar y simplificar $y = \ln[\coth(5x) - \operatorname{csch}(5x)]$
Solución: *si* $y = \ln[\coth(5x) - \operatorname{csch}(5x)]$

$$entonces \quad y' = \frac{[-5\operatorname{csch}^2(5x) + 5\operatorname{csch}(5x)\coth(5x)]}{\coth(5x) - \operatorname{csch}(5x)}$$

$$y' = \frac{5[-\operatorname{csch}^2(5x) + \operatorname{csch}(5x)\coth(5x)]}{\coth(5x) - \operatorname{csch}(5x)} = \frac{5[\coth(5x) - \operatorname{csch}(5x)]\operatorname{csch}(5x)}{\coth(5x) - \operatorname{csch}(5x)} = 5\operatorname{csch}(5x)$$

$$y = \ln[\coth(5x) - \operatorname{csch}(5x)] \ \Rightarrow \ y' = \ 5\operatorname{csch}(5x)$$

10. $-$Derivar y simplificar $\operatorname{arctanh} y = \arctan(\operatorname{sech} x)$

Solución: $u = \operatorname{sech} x \quad , \quad u' = \operatorname{sech} x \tanh x$

$$\operatorname{arctanh} y = \arctan(u) \quad \Rightarrow \quad \frac{y'}{1-y^2} = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

$$\frac{y'}{1-y^2} = -\frac{\operatorname{sech} x \tanh x}{1+\operatorname{sech}^2 x} \quad \Rightarrow \quad y' = -(1-y^2)\frac{\operatorname{sech} x \tanh x}{1+\operatorname{sech}^2 x}$$

11. $-$Derivar y simplificar $y = \ln^3\left(\tanh\left(e^{x^2}\right)\right)$

Solución: $y = 3\ln^2\left(\tanh\left(e^{x^2}\right)\right).\dfrac{2x}{\tanh\left(e^{x^2}\right)}\ .\operatorname{sech}^2\left(e^{x^2}\right).\,e^{x^2}$

$$y' = \frac{6x\ ln^2\left(\tanh\left(e^{x^2}\right)\right).\operatorname{sech}^2\left(e^{x^2}\right)e^{x^2}}{\tanh\left(e^{x^2}\right)}$$

12. $-$Derivar y simplificar $y = \operatorname{arccosh}\left(e^{x^2}\right)$

Solución: $\quad si\ \ u = e^{x^2}, \quad \dfrac{du}{dx} = 2x\ e^{x^2}$

$$y = \operatorname{arccosh}\left(e^{x^2}\right) \quad \Rightarrow \quad y = \operatorname{arcosh}(u), \quad luego \quad y' = \frac{1}{\sqrt{u^2-1}} \cdot \frac{du}{dx}$$

$$es\ decir\ \ y' = \frac{2x\ e^{x^2}}{\sqrt{e^{2x^2}-1}}$$

13. $-$Derivar y simplificar $y = \ln(\ln(\operatorname{arcsech}(x^2)))$

Solución: $u = \operatorname{arcsech}(x^2) \quad \Rightarrow \quad u' = \dfrac{2x}{x^2\sqrt{1-x^4}}$

$$y = \ln(\ln(\operatorname{arcsech}(x^2))) \quad \Rightarrow \quad y = \ln(\ln u) \quad \Rightarrow \quad y' = \frac{1}{\ln(u)} \cdot \frac{1}{u} \cdot u'$$

$$y' = \frac{2x}{\ln(\operatorname{arcsech}(x^2))\ .\operatorname{arcsech}(x^2)\ .x^2\sqrt{1-x^4}}$$

$$y' = \frac{2}{x\sqrt{1-x^4}\ .\ln(\operatorname{arcsech}(x^2))\operatorname{arcsech}(x^2)}$$

14. −Derivar y simplificar $y = \ln[\text{arcsenh}(e^{2x})]^{3/2}$

Solución: $y = \ln[\text{arcsenh}(e^{2x})]^{3/2} \quad \Rightarrow \quad y' = \dfrac{\frac{3}{2}[\text{arcsenh}(e^{2x})]^{1/2}}{[\text{arcsenh}(e^{2x})]^{3/2}} \cdot \dfrac{2\,e^{2x}}{\sqrt{e^{4x}+1}}$

$$y' = \frac{3\,e^{2x}}{\text{arcsenh}(e^{2x})\,.\sqrt{e^{4x}+1}}$$

15. −Derivar y simplificar $y = [\text{csch}^2(x^2-3x+1)]^{1/5}$

Solución: $si\ \ u = x^2 - 3x + 1 \quad y \quad u' = 2x - 3 \quad entonces \quad y = [\text{csch}\,(u)]^{2/5}$

$$y' = \frac{2}{5}[\text{csch}\,(u)]^{-\frac{3}{5}}[-\text{csch}(u)\coth(u)]\left(\frac{du}{dx}\right) \ \Rightarrow\ y' = -\frac{2}{5}[\text{csch}\,(u)]^{\frac{2}{5}}[\coth(u)]u'$$

$$y' = -\frac{2}{5}\text{csch}^{\frac{2}{5}}(x^2-3x+1)\coth(x^2-3x+1)\cdot(2x-3)$$

16. −Derivar y simplificar $y = [\text{sech}^3(3x).\cosh^3(3x)]$

Solución: $\text{sech}^3(u) = \dfrac{1}{\cosh^3(u)}$

$$y = [\text{sech}^3(3x).\cosh^3(3x)] \ \Rightarrow\ y = \frac{1}{\cosh^3(3x)}\,.\cosh^3(3x)$$

$$y = 1 \quad \Rightarrow\ y' = 0$$

17. −*Derivar y simplificar* $y = \dfrac{1}{x}\text{sech}\left(\dfrac{1}{x}\right)$

Solución: $y = \dfrac{1}{x}\,.\text{sech}\left(\dfrac{1}{x}\right) \ \Rightarrow\ y' = \dfrac{-1}{x^2}\text{sech}\left(\dfrac{1}{x}\right) + \dfrac{1}{x}\left[-\text{sech}\left(\dfrac{1}{\text{x}}\right)\tanh\left(\dfrac{1}{x}\right).\left(-\dfrac{1}{x^2}\right)\right]$

$$y' = -\frac{1}{x^2}\text{sech}\left(\frac{1}{x}\right)\left[1 - \frac{\tanh\left(\frac{1}{x}\right)}{x}\right]$$

18. −Derivar y simplificar $3\,\text{arcsenh}(x^2+y^2) = \text{arctanh}\,x$

Solución: $3.\dfrac{2x + 2\,y\,y'}{\sqrt{(x^2+y^2)^2+1}} = \dfrac{1}{1-x^2} \quad \Rightarrow \quad 2x + 2yy' = \dfrac{\sqrt{(x^2+y^2)^2+1}}{3(1-x^2)}$

$$2yy' = \frac{\sqrt{(x^2+y^2)^2+1}}{3(1-x^2)} - 2x \quad \Rightarrow \quad y' = \frac{\sqrt{(x^2+y^2)^2+1}}{6y(1-x^2)} - \frac{x}{y}$$

19. $-$Derivar y simplificar $y = \ln\left(\operatorname{senh}\left(\sqrt{x}\right)\right)$

Solución: $u = \sqrt{x} \quad \Rightarrow \quad \dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}}$

$$y = \ln\left(\operatorname{senh}\left(\sqrt{x}\right)\right) \;\; \Rightarrow \;\; y = \ln(\operatorname{senh} u)$$

$$y' = \frac{1}{\operatorname{senh}(u)} \cdot \cosh(u) \cdot \frac{du}{dx} \quad \Rightarrow \quad y' = \frac{\cosh\left(\sqrt{x}\right)}{2\sqrt{x}\operatorname{senh}\left(\sqrt{x}\right)} = \frac{1}{2\sqrt{x}}.\coth\left(\sqrt{x}\right)$$

20. $-$*Derivar y simplificar* $y = \dfrac{1}{\sqrt{\tanh\left(\ln^2(3x)\right)}}$

Solución: *si* $u = 3x$ *entonces* $\dfrac{du}{dx} = 3$

por lo tanto $y = \dfrac{1}{\sqrt{\tanh\left(\ln^2(u)\right)}} \;\; \Rightarrow \;\; y = \left[\tanh\left(\ln^2(u)\right)\right]^{-1/2}$

$$y' = -\frac{1}{2}\left[\tanh\left(\ln^2(u)\right)\right]^{-3/2}\operatorname{sech}^2\left(\ln^2(u)\right)2\ln(u)\,.\frac{1}{u}\cdot\frac{du}{dx}$$

$$y' = -\frac{1}{2}\left[\tanh\left(\ln^2(3x)\right)\right]^{-3/2}\operatorname{sech}^2\left(\ln^2(3x)\right)2\ln(3x)\,.\frac{1}{3x}\cdot 3$$

$$y' = -\frac{1}{x}\left[\tanh\left(\ln^2(3x)\right)\right]^{-3/2}\operatorname{sech}^2\left(\ln^2(3x)\right)\ln(3x)$$

Reglas de Integración

$$1.- \int (du + dv + dw) \ = \int du + \int dv + \int dw$$

La integral de una suma de funciones es igual a la suma de las integrales de las funciones sumando.

$$2.- \int adu = a \int du.$$

La integral del producto de una constante por una función es igual a la constante por la integral de la función.

$$3.- \int du = u + C \ .$$

La integral de la diferencial de u*es igual a la función u más la constante de integración*

$$4.- \int u^n du = \ \frac{u^{n+1}}{n+1} + \ C \ .$$

La integral de la potencia enésima de la función u*es igual a la potencia en* $(n+1)$ *de* "u" *dividida por* $(n+1)$.

$$5.- \int \frac{du}{u} \ = \ \ln u + C \ .$$

$(n = -1$ *en el número cuatro*$)$.

$$6.- \int e^u du = \ e^u + C \ .$$

La integral de una potencia de base "e"*es igual a la misma potencia más la constante de integración.*

$$7.- \int a^u du = \frac{a^u}{\ln a} + C \ .$$

La integral de una potencia de base "a"*es igual al cociente de la potencia dividida por el logaritmo natural de la base más la constante de integración.*

Reglas de Integración para funciones trigonométricas

$1.- \int \operatorname{sen} u \, du = \ \cos u + C$ $2.- \int \cos u \, du = \ -\operatorname{sen} u + C$

$3.- \int \sec^2 u \, du = \ \tan u + C$ $4.- \int \csc^2 u \, du = \ -\cot u + C$

$5.- \int \sec u \tan u \, du = \sec u + C$

$6.- \int \csc u \cot u \, du = -\csc u + C$

$7.- \int \tan u \, du = \ln(\sec u) + C$

$8.- \int \cot u \, du = \ln(\operatorname{sen} u) + C$

$9.- \int \frac{du}{u^2 + a^2} = \frac{1}{a}\arctan(u) + C$

$10.- \int \frac{du}{\sqrt{a^2 - u^2}} = \operatorname{arcsen}\left(\frac{u}{a}\right) + C$

$11.- \int \frac{du}{\sqrt{u^2 - a^2}} = \ln\left(u + \sqrt{u^2 - a^2}\right) + C$

$12.- \int \frac{du}{\sqrt{u^2 + a^2}} = \ln\left(u + \sqrt{u^2 + a^2}\right) + C$

$13.- \int \sec u \, du = \ln(\sec u + \tan u) + C$

$14.- \int \csc u \, du = \ln(\csc u - \cot u) + C$

CALCULAR LAS SIGUIENTES INTEGRALES.

$1.- Resolver\colon \int (\sqrt{x} + 8x - 3)dx$

$Solución\colon \int (\sqrt{x} + 8x - 3)dx = \int x^{1/2}\, dx + 8\int x dx - 3\int dx$

$$\int (\sqrt{x} + 8x - 3)dx = \frac{2}{3}x^{3/2} + 4x^2 - 3x + c$$

$2.- Resolver \int (2\sqrt{x} - \sqrt[3]{x} - x^4)dx$

$Solución\colon \int (2\sqrt{x} - \sqrt[3]{x} - x^4)dx = 2\int x^{1/2}\, dx - \int x^{1/3}\, dx - \int x^4\, dx$

$$2.\frac{2}{3}x^{3/2} - \frac{3}{4}x^{4/3} - \frac{1}{5}x^5 + c = \frac{4}{3}x^{3/2} - \frac{3}{4}x^{4/3} - \frac{1}{5}x^5 + c$$

$$\int (2\sqrt{x} - \sqrt[3]{x} - x^4)dx = \frac{4}{3}x^{3/2} - \frac{3}{4}x^{4/3} - \frac{1}{5}x^5 + c$$

$3.-Resolver \int \frac{2dx}{x^3}$

$Solución\colon \int \frac{2dx}{x^3} = 2\int \frac{dx}{x^3} + c = \int x^{-3}\,.\,dx + c$

$$2\int \frac{dx}{x^3} = 2\cdot\frac{x^{-2}}{-2} + c = -\frac{1}{x^2} + c$$

$4.-Resolver \ \int \frac{dx}{\sec(x)}$

$Solución: \int \frac{dx}{\sec(x)} = \int \frac{dx}{\frac{1}{\cos x}} = \int \cos x \, dx = \operatorname{sen} x + c$

$5.-Resolver: \int [\sqrt{x}\,.(x+\sqrt[5]{x})]dx$

$Solución: \int [\sqrt{x}\,.(x+\sqrt[5]{x})]dx = \int (\sqrt{x}\ .x)\,dx + \int (\sqrt{x}\,\sqrt[5]{x}\,)dx \ + c$

$$\int \left(x^{\frac{1}{2}}.x\right)dx + \int (x^{\frac{1}{2}}.x^{\frac{1}{5}})dx + c = \int x^{\frac{3}{2}}\,dx + \int x^{\frac{7}{10}}\,dx \ + c$$

$$\int [\sqrt{x}\,.(x+\sqrt[5]{x})]dx \ = \frac{2}{5}x^{\frac{5}{2}} + \frac{10}{17}x^{\frac{17}{10}} + c$$

$6.-Resolver \ \int \frac{(x^2+2x)^2dx}{x^2}$

$Solución: \int \frac{(x^2+2x)^2dx}{x^2} = \int \frac{(x^4+4x^3+4x^2)dx}{x^2} + c$

$$\int (x^2+4x+4)dx + c = \int x^2\,dx + 4\int xdx + 4\int dx + c$$

$$\int \frac{(x^2+2x)^2dx}{x^2} = \frac{1}{3}x^3 + 2x^2 + 4x + c$$

$7.-Resolver: \int x.\sqrt{x^2+2}\,dx$

$Solución: \qquad u = x^2+2 \quad \Rightarrow \quad \frac{du}{2} = xdx$

$$\int \left(x.\sqrt{x^2+2}\right)dx = \frac{1}{2}\int u^{1/2}\,du = \frac{1}{3}u^{3/2} + c = \frac{1}{3}(x^2+2)^{3/2} + c$$

$8.-Resolver: \int \tan(x)dx$

$Solución: \int \tan(x)dx = \int \frac{\text{sen}(x)dx}{\cos(x)}$

$$u = \cos(x) \quad \Rightarrow \quad -du = \text{sen}(x)dx$$

$$\int \tan(x)dx = -\int \frac{du}{u} = -\ln(u) + c = \ln\big(\cos(x)\big) + c = \ln(1) - \ln\big(\cos(x)\big) + c$$

$$\int \tan(x)dx \ = \ln(\sec(x) + c$$

$9.-Resolver \int \frac{(x^7 + 5x^5 - 6x^3 + 8)dx}{x^2}$

$$Solución: \int \frac{(x^7 + 5x^5 - 6x^3 + 8)dx}{x^2} = \int x^5\, dx + 5\int x^3\, dx - 6\int xdx + 8 \int x^{-2}\, dx + c$$

$$\int \frac{(x^7 + 5x^5 - 6x^3 + 8)dx}{x^2} = \frac{1}{6}x^6 + \frac{5}{4}x^4 - 3x^2 - \frac{8}{x} + c$$

$10.- Resolver: \int (4x+3)^2\, dx$

$Solución: \int (4x+3)^2\, dx \ = \int 16x^2\, dx + \int 24x + \int 9dx + c$

$$\int (4x+3)^2\, dx = \frac{16}{3}x^3 + 12x^2 + 9x + c$$

$11.-Resolver \int \frac{x^2dx}{x^3 - 2}$

$Solución: \ u = \ x^3 - 2 \quad \Rightarrow \frac{du}{3} = \ x^2dx$

$$\int \frac{x^2dx}{x^3-2} = \frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\ln(u) + c = \frac{1}{3}\ln(x^3-2) + c$$

$12.- Resolver: \int 2x.e^{x^2}dx$

$Solución: \ u = \ x^2 \quad \rightarrow \quad du = 2x\, dx$

$$\int 2x.e^{x^2}dx = \int e^{x^2}.2xdx$$

$$\int e^u\,du = e^u + c = e^{x^2} + c$$

$13.-Resolver\ \int \frac{dx}{1-x}$

$Solución$: $u = 1-x \quad \Rightarrow -du = dx$

$$\int \frac{dx}{1-x} = \int -\frac{du}{u} + c = -\int \frac{du}{u} + c = -\ln u + c = -\ln(1-x) + c$$

$14.-Resolver\ \int \frac{(x+2)dx}{2\sqrt{x+2}}$

$Solución$: $u = x+2 \quad \rightarrow \quad du = dx$

$$\int \frac{(x+2)dx}{2\sqrt{x+2}} = \frac{1}{2}\int \frac{udu}{u^{1/2}} = \frac{1}{2}\int u.u^{-1/2}\,du = \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}.\frac{2}{3}u^{3/2} + c = \frac{1}{3}u^{3/2} + c$$

$$\int \frac{(x+2)dx}{2\sqrt{x+2}} = \frac{1}{3}\,(x+2)^{3/2} + c$$

$15.-Resolver$: $\int (3x^2-1)^3dx$

$Solución$: $\int (3x^2-1)^3\,dx = 27\int x^6 - 27\int x^4dx + 9\int x^2dx - \int dx$

$$\int (3x^2-1)^3dx = \frac{27}{7}x^7 - \frac{27}{5}x^5 + 3x^3 - x + c$$

$16.-\ Resolver$: $\int \left(\frac{5}{x} - \frac{x}{5}\right)dx$

$Solución$: $\int \left(\frac{5}{x} - \frac{x}{5}\right)dx = \int \frac{5dx}{x} - \frac{1}{5}\int xdx + c = 5\ln(x) - \frac{1}{10}x^2 + c$

$17.-Resolver$: $\int \cos(x^2)2x\,dx$

$Solución$: $\quad u = x^2 \quad \rightarrow \quad du = 2x\,dx$

$$\int \cos(x^2)2x\,dx = \int \cos u du = \operatorname{sen} u + c = \operatorname{sen}(x^2) + c$$

$18. - Resolver$: $\int (1 - \cos\ x)^3 \operatorname{sen} x\,dx$

$Solución$: $\int (1 - \cos x)^3 \operatorname{sen} x\,dx = \int \big(1 - 3\cos x + 3\cos^2 x - \cos^3(x)\big)\operatorname{sen}(x)dx$

$$= \int \operatorname{sen}(x)dx - 3\int \cos x \operatorname{sen} x\,dx + 3\int \cos^2(x)\operatorname{sen} x\,dx - \int \cos^3(x)\operatorname{sen}(x)dx$$

$$\int (1 - \cos x)^3 \operatorname{sen} x\,dx = -\cos(x) + \frac{3}{2}\cos^2(x) - \cos^3(x) + \frac{1}{4}\cos^4(x) + C$$

$19. -Resolver \int \frac{x^2dx}{x^2 - 2}$

$Solución$: $\int \frac{x^2dx}{x^2 - 2} = \int \left(1 + \frac{2}{x^2 - 2}\right)dx = \int dx + 2\int \frac{dx}{x^2 - 2} = x + \frac{1}{\sqrt{2}}\ln\frac{x - \sqrt{2}}{x + \sqrt{2}} + c$

$20. - Resolver$: $\int (3x^2 - 1)(x + 2)dx$

$Solución$: $\int (3x^2 - 1)(x + 2)dx = \int (3x^3 - x + 6x^2 - 2)dx$

$$= 3\int x^3dx - \int xdx + 6\int x^2\,dx - 2\int dx = \frac{3}{4}x^4 - \frac{1}{2}x^2 + 2x^3 - 2x + c$$

$21. -Resolver \int \frac{(5x + 3)^2dx}{4}$

$Solución$: $\int \frac{(5x + 3)^2dx}{4} = \int \frac{(25x^2 + 30x + 9)dx}{4} + c$

$$\int \left(\frac{25}{4}x^2 + \frac{30}{4}x + \frac{9}{4}\right)dx + c = \frac{25}{4}\int x^2dx + \frac{15}{2}\int xdx + \frac{9}{4}\int dx + c$$

$$\int \frac{(5x + 3)^2dx}{4} = \frac{25}{4}.\frac{1}{3}x^3 + \frac{30}{4}.\frac{1}{2}x^2 + \frac{9}{4}x + c = \frac{25}{12}x^3 + \frac{15}{4}x^2 + \frac{9}{4}x + c$$

$22.-\ Resolver$: $\displaystyle\int (5x^3-1)^2 dx$

$Solución$: $\displaystyle\int (5x^3-1)^2 dx = \int (25x^6-10x^3+1)dx$

$$\int (5x^3-1)^2 dx\ = 25\int x^6\,dx - 10\int x^3\,dx + \int dx = 25.\frac{1}{7}x^7 - 10.\frac{1}{4}x^4 + x + c$$

$$\int (5x^3-1)^2 dx\ = \frac{25}{7}x^7 - \frac{5}{2}x^4 + x + c$$

$23.-\ Resolver$: $\displaystyle\int e^{5x+3}\,dx$

$Solución$: $u = 5x+3 \ \rightarrow \ \dfrac{du}{5} = dx$

$$\int e^{5x+3}\,dx = \frac{1}{5}\int e^u\,.\,du = \frac{1}{5}.\,e^u + c = \frac{1}{5}.\,e^{5x+3} + c$$

24.-$Resolver$: $\int (2x^2+1)^{\frac{1}{2}}x^3 dx$

$Solución$: $u = 2x^2+1 \ \rightarrow \ \dfrac{du}{4} = xdx \ ,\quad x^2 = \dfrac{u-1}{2}$

$$\int (2x^2+1)^{\frac{1}{2}}x^3 dx = \int x^2\,(2x^2+1)^{\frac{1}{2}}\,xdx = \int \left(\frac{u-1}{2}\right)u^{\frac{1}{2}}\frac{du}{4} = \frac{1}{8}\int (u-1)u^{\frac{1}{2}}\,du$$
$$= \frac{1}{8}\int u^{\frac{3}{2}}\,du - \frac{1}{8}\int u^{\frac{1}{2}}\ du$$

$$\rightarrow\ \frac{1}{20}u^{\frac{5}{2}} - \frac{1}{12}u^{\frac{3}{2}} + c = \frac{1}{20}(2x^2+1)^{\frac{5}{2}} - \frac{1}{12}(2x^2+1)^{\frac{3}{2}} + c$$

$25.-Resolver$: $\displaystyle\int \frac{(x+3)dx}{(3-x)^{2/3}}$

$Solución$: $u = 3-x, \qquad -du = dx \qquad x+3 = 6-u$

$$\int \frac{(x+3)dx}{(3-x)^{2/3}} = \int \frac{(6-u)(-du)}{u^{2/3}} = -6\int u^{-\frac{1}{2}}du + \int u^{+\frac{1}{3}}\,du$$

$$= -18u^{1/3} + \frac{3}{4}u^{4/3} + c = -18(3-x)^{1/3} + \frac{3}{4}(3-x)^{4/3} + c$$

$26.-$ *Resolver*: $\int (1+\operatorname{sen}(2x))^{3/2}.\cos(2x)dx$

Solución: $u = 1 + \operatorname{sen}(2x) \quad \rightarrow \quad \frac{du}{2} = \cos(2x)dx$

$$\frac{1}{2}\int u^{\frac{3}{2}}\,du = \frac{1}{2}.\frac{2}{5}u^{5/2} + c = \frac{1}{5}u^{5/2} + c = \frac{1}{5}(1+\operatorname{sen}(2x))^{5/2} + c$$

27. $-$*Resolver*: $\int \frac{\operatorname{sen} x dx}{\cos^2 x}$

Solución: $u = \cos x \quad \rightarrow \quad -du = \operatorname{sen} x\, dx$

$$\int \frac{\operatorname{sen} x\, dx}{\cos^2 x} = \int \cos^{-2} x \operatorname{sen} x\, dx = -\int u^{-2}\, du = u^{-1} + c = \cos(x)^{-1} + c = \frac{1}{\cos(x)} + c$$
$$= \sec(x) + c$$

$28.-$*Resolver*: $\int (x^{-2}+x)\,(1-2x^{-3})dx$

Solución: $u = x^{-2} + x \quad \rightarrow \quad du = (-2x^{-3}+1)dx \quad \rightarrow \quad du = (1-2x^{-3})dx$

$$\int (x^{-2}+x)\,(1-2x^{-3})dx = \int u(du) = \int udu = \frac{u^2}{2} + c = \frac{1}{2}(x^{-2}+x)^2 + c$$

$29.-$*Resolver* $\int \frac{xdx}{(1-2x^2)^3}$

Solución: $u = 1 - 2x^2 \rightarrow -\frac{du}{4} = xdx$

$$\int \frac{xdx}{(1-2x^2)^3} = \frac{-1}{4}\int \frac{du}{u^3} = \frac{-1}{4}\int u^{-3}\,du = \frac{-1}{4}.\frac{1}{-2}u^{-2} + c = \frac{1}{8}(-2x^2+1)^{-2} + c$$

$30.-$*Resolver* $\int \frac{(x^2+x)dx}{4-3x^2-2x^3}$

Solución: $u = 4 - 3x^2 - 2x^3 \quad \rightarrow \quad du = -6(x^2+x)dx) \quad \rightarrow \quad -\frac{du}{6} = (x^2+x)\,dx$

$$\int \frac{(x^2+x)dx}{4-3x^2-2x^3} == -\frac{1}{6}\int \frac{du}{u} = -\frac{1}{6}\ln\left(4-3x^2-2x^3\right)+c$$

$31. - Resolver \int \sqrt{x^4+x^2}\,(10x^3+5x)\,dx$

$Solución: u = x^4+x^2 \rightarrow \frac{5du}{2} = \left(10x^3+5x\right)dx$

$$\int \sqrt{x^4+x^2}\,(10x^3+5x)\,dx = \frac{5}{2}\int u^{\frac{1}{2}}\,du = \frac{5}{3}u^{\frac{3}{2}} = \frac{5}{3}(\sqrt{x^4+x^2}^{3} + c$$

$32. - Resolver \int \frac{dx}{\sqrt{x}(\sqrt{x}-3)^5}$

$Solución: u = \sqrt{x}-3 \rightarrow 2du = \frac{dx}{\sqrt{x}}$

$$\int \frac{dx}{\sqrt{x}(\sqrt{x}-3)^5} = 2\int \frac{du}{u^5} = -\frac{1}{2u^4} + c = -\frac{1}{2\left(\sqrt{x}-3\right)^4} + c$$

$33. - Resolver \int \left(1+\frac{1}{x}\right)\frac{1}{x^2}dx$

$Solución: u = \left(1+\frac{1}{x}\right) \rightarrow -du = \frac{dx}{x^2}$

$$\int \left(1+\frac{1}{x}\right)\frac{1}{x^2}dx = -\int udu = -\frac{u^2}{2} + c = -\frac{1}{2}\left(1+\frac{1}{x}\right)^2 + c$$

$34. - Resolver \int \frac{(\sqrt{x}-1)^{\frac{3}{2}}\,dx)}{\sqrt{x}}$

$Solución: u = \sqrt{x}-1 \rightarrow 2du = \frac{dx}{\sqrt{x}}$

$$\int \frac{(\sqrt{x}-1)^{\frac{3}{2}}\,dx)}{\sqrt{x}} = 2\int u^{\frac{3}{2}}\,du = \frac{4}{5}u^{\frac{5}{2}} + c$$

$$\int \frac{(\sqrt{x}-1)^{\frac{3}{2}}\,dx)}{\sqrt{x}} = \frac{4}{5}\left(\sqrt{x}-1\right)^{5/2} + c$$

35. *Resolver* $\int \frac{\left(x^{\frac{1}{3}}-2\right)^5}{\sqrt[3]{x^2}}dx$

Solución: $u = x^{\frac{1}{3}} - 2 \ \ , \ \ 3du = \frac{dx}{\sqrt[3]{x^2}}$

$$\int \frac{\left(x^{\frac{1}{3}}-2\right)^5}{\sqrt[3]{x^2}}dx = 3\int u^5du = \frac{1}{2}u^6 + c = \frac{1}{2}\left(x^{\frac{1}{3}}-2\right)^6 + c$$

36. −*R*esolver: $\int x^2 \,.\, (x+1)^{1/2}\, dx$

Solucion: $u = x+1 \ \rightarrow \ (u-1)^2 = x^2 \ , \quad du = dx$

$$\int (u-1)^2\, u^{\frac{1}{2}}\, du = \int (u^2-2u+1)u^{\frac{1}{2}}\, du = \int u^{\frac{5}{2}}\, du - 2\int u^{\frac{3}{2}}\, du + \int u^{\frac{1}{2}}\, du$$
$$= \frac{2}{7}u^{\frac{7}{2}} - 2\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}} + c = \frac{2}{7}u^{\frac{7}{2}} - \frac{4}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}} + c$$
$$= \frac{2}{7}(x+1)^{\frac{7}{2}} - \frac{4}{5}(x+1)^{\frac{5}{2}} + \frac{2}{3}(x+1)^{\frac{3}{2}} + c$$

37. −*Resolver* $\int \frac{3xdx}{(1-3x)^2}$

Solución: $\mathrm{u} = 1 - 3\mathrm{x} \ \rightarrow \ -du = 3dx \ \ , \ \ x = \frac{1-u}{3}$

$$\int \frac{3xdx}{(1-3x)^2} = -\frac{1}{3}\int \frac{(1-u)du}{u^2} = -\frac{1}{3}\int (1-u)(u^{-2})du = -\frac{1}{3}\int (u^{-2}-u^{-1})du$$
$$= -\frac{1}{3}\left[\int u^{-2}du - \int u^{-1}du\right] = -\frac{1}{3}[-u^{-1} - \ln(u)] + c$$

$$\int \frac{3xdx}{(1-3x)^2} = \frac{1}{3}(1-3x)^{-1} + \frac{1}{3}\ln(1-3x) + c$$

38. − *Resolver*: $\int (3-x)^{\frac{5}{2}}\, xdx$

Solución: $u = 3 - x \ \ , \ \ x = 3 - u \quad -du = dx$

$$\int (3-x)^{\frac{5}{2}}\,x dx = \int u^{\frac{5}{2}}(3-u)(-du) = \int u^{\frac{5}{2}}\,(u-3)du = \int u^{\frac{7}{2}}du - 3\int u^{\frac{5}{2}}\,du$$
$$= \int u^{\frac{7}{2}}\,du - 3\int u^{\frac{5}{2}}\,du = \frac{2}{9}u^{9/2} - 3.\frac{2}{7}u^{7/2} + c = \frac{2}{9}u^{9/2} - \frac{6}{7}u^{7/2} + c$$
$$= \frac{2}{9}(-x+3)^{9/2} - \frac{6}{7}(-x+3)^{7/2} + c$$

$39. -Resolver \int \sqrt[3]{3+x}\,(x+1)^2 dx$

$Solución$: $u = 3 + x \;\rightarrow x + 1 = u - 2 \;\rightarrow\; du = dx$

$$\int \sqrt[3]{3+x}\,(x+1)^2 dx = \int \sqrt[3]{u}\,(u-2)^2 du = \int \sqrt[3]{u}\,(u^2 - 4u + 4)du$$

$$\int \sqrt[3]{u}\,(u^2 - 4u + 4)du = \int u^{\frac{7}{3}}du - 4\int u^{\frac{4}{3}}du + 4\int u^{\frac{1}{3}}\,du = \frac{3}{10}u^{\frac{10}{3}} - \frac{12}{7}u^{\frac{7}{3}} + 3u^{\frac{4}{3}} + c$$

$$\int \sqrt[3]{3+x}\,(x+1)^2 dx = \frac{3}{10}(3+x)^{\frac{10}{3}} - \frac{12}{7}(3+x)^{\frac{7}{3}} + 3(3+x)^{\frac{4}{3}} + c$$

$40. -Resolver \int x^{-3}\,(1 + \frac{1}{2x})^{\frac{3}{2}}dx$

$Solución$: $u = 1 + \frac{1}{2x} \;\rightarrow\; u - 1 = \frac{1}{2}x^{-1} \;\rightarrow\; 2(u-1) = x^{-1} \;\rightarrow\; -2du = x^{-2}dx$

$$\int x^{-1}\,(1+\frac{1}{2x})^{\frac{3}{2}}x^{-2}dx = \int 2(u-1)(u)^{\frac{3}{2}}.(-2du) = -4\int (u-1)(u)^{\frac{3}{2}}\,du$$
$$= -4\int \left[\left(uu^{\frac{3}{2}} - u^{\frac{3}{2}}\right)du\right] = -4\int u^{\frac{5}{2}}\,du + 4\int u^{\frac{3}{2}}du = -4.\frac{2}{7}u^{\frac{7}{2}} + 4.\frac{2}{5}u^{\frac{5}{2}} + c$$
$$= -\frac{8}{7}u^{\frac{7}{2}} + \frac{8}{5}u^{\frac{5}{2}} + c = -\frac{8}{7}(1+\frac{1}{2x})^{\frac{7}{2}} + \frac{8}{5}(1+\frac{1}{2x})^{\frac{5}{2}} + c$$

$41. -Resolver \int \frac{(x+1)dx}{\sqrt{x^2+2x+2}}$

$Solución$: $u = x^2 + 2x + 2 \;\rightarrow\; du = 2x + 2dx \;\rightarrow\; \frac{du}{2} = (x+1)dx$

$$\int \frac{(x+1)dx}{\sqrt{x^2+2x+2}} = \frac{1}{2}\int \frac{du}{u^{1/2}} = \frac{1}{2}\int u^{\frac{-1}{2}}\,du = \frac{1}{2}.2\,u^{\frac{1}{2}} + c = u^{\frac{1}{2}} + c = (x^2+2x+2)^{\frac{1}{2}} + c$$

$42.-Resolver \int \frac{(x+3)dx}{\sqrt[3]{(9-x)^2}}$

$Solución: \quad u = 9 - x \;\; \rightarrow \;\; x + 3 = 12 - u \;\; \rightarrow -du = dx$

$$\int \frac{(x+3)dx}{\sqrt[3]{(9-x)^2}} = \int \frac{(12-u)(-du)}{u^{\frac{2}{3}}} = 12\int u^{-\frac{2}{3}}(-du) + \int u^{\frac{1}{3}}du = -36u^{\frac{1}{3}} + \frac{3}{4}u^{\frac{4}{3}} + c$$

$$\int \frac{(x+3)dx}{\sqrt[3]{(9-x)^2}} = -36(9-x)^{\frac{1}{3}} + \frac{3}{4}(9-x)^{\frac{4}{3}} + c$$

$43.-Resolver: \quad \int \left(x+\frac{1}{x}\right)^{\frac{5}{3}} \left(\frac{x^2-1}{x^2}\right) dx$

$Solución: \quad u = x + \frac{1}{x} \;\; \rightarrow \;\; du = \frac{x^2-1}{x^2}dx$

$$\int \left(x+\frac{1}{x}\right)^{\frac{5}{3}} \left(\frac{x^2-1}{x^2}\right) dx = \int u^{\frac{5}{3}}\,du = \frac{3}{8}u^{\frac{8}{3}} + c = \frac{3}{8}\left(x+\frac{1}{x}\right)^{\frac{8}{3}} + c$$

$44.-Resolver \int \sqrt[3]{2x^2-1}x^3dx$

$Solución: \; u = 2x^2 - 1 \;\; \rightarrow \;\; x^2 = \frac{u+1}{2} \;\; \rightarrow \frac{du}{4} = xdx$

$$\int \sqrt[3]{2x^2-1}x^3dx = \int x^2\sqrt[3]{2x^2-1}x\,dx = \frac{1}{4}\int \left(\frac{u+1}{2}\right)\sqrt[3]{u}\,du = \frac{1}{8}\int u^{\frac{4}{3}}\,du + \frac{1}{8}\int u^{\frac{1}{3}}\,du$$

$$\int \sqrt[3]{2x^2-1}x^3dx = \frac{3}{56}u^{\frac{7}{3}} + \frac{3}{32}u^{\frac{4}{3}} + c = \frac{3}{56}(2x^2-1)^{\frac{7}{3}} + \frac{3}{32}(2x^2-1)^{\frac{4}{3}} + c$$

$45.-Resolver \int \frac{1}{x^2}\left(\frac{x-1}{x}\right)^{\frac{1}{2}} dx$

$Solución: \; u = 1 - \frac{1}{x} \;\; \rightarrow \;\; du = \frac{dx}{x^2}$

$$\int \frac{1}{x^2}\left(\frac{x-1}{x}\right)^{\frac{1}{2}} dx = \int u^{\frac{1}{2}}du = \frac{2}{3}u^{\frac{3}{2}} + c = \frac{2}{3}\left(\frac{x-1}{x}\right)^{\frac{3}{2}} + c$$

$46.-Resolver\ \int x^3\sqrt{2x^2+3}dx$

$Solución$: $\int x^3\sqrt{2x^2+3}\,dx = \int x^2\sqrt{2x^2+3}\,x\,dx$

$$u = 2x^2+3 \ \rightarrow\ \frac{du}{4} = xdx \ \rightarrow\ \frac{u-3}{2} = x^2$$

$$\int x^2\sqrt{2x^2+3}\,x\,dx = \int \left(\frac{u-3}{2}\right)u^{\frac{1}{2}}\frac{du}{4} = \frac{1}{8}\int u^{\frac{3}{2}}\,du - \frac{3}{8}\int u^{\frac{1}{2}}\,du = \frac{1}{20}u^{\frac{5}{2}} - \frac{1}{4}u^{\frac{3}{2}} + c$$

$$\int x^2\sqrt{2x^2+3}\,x\,dx = \frac{1}{20}(2x^2+3)^{\frac{5}{2}} - \frac{1}{4}(2x^2+3)^{\frac{3}{2}} + c$$

47.- $Resolver$: $\int x(x^2+2)^{10}dx$

$Solución$: $u = x^2+2 \ \rightarrow\ \frac{du}{2} = xdx$

$$\int x(x^2+2)^{10}dx = \frac{1}{2}\int u^{10}du = \frac{1}{2}.\frac{1}{11}u^{11} + c = \frac{1}{22}u^{11} + c = \frac{1}{22}\ (x^2+2)^{11} + c$$

$48.-Resolver$: $\int \frac{(2x+3)dx}{\sqrt{x^2+3x}}$

$Solución$: $u = x^2+3x \ \rightarrow\ du = (2x+3)dx$

$$\int \frac{(2x+3)dx}{\sqrt{x^2+3x}} = \int \frac{(2x+3)dx}{(x^2+3x)^{1/2}} = \int \frac{du}{u^{1/2}} = \int u^{-1/2}\,du = 2u^{1/2} + c = 2(x^2+3x)^{1/2} + c$$

$49.-Resolver\ \int \sqrt{x}\sqrt{1+x\sqrt{x}}\,dx$

$Solución$: $u = 1+x\sqrt{x} \ \rightarrow\ u = 1+x^{\frac{3}{2}} \ \rightarrow\ \frac{2}{3}du = \sqrt{x}dx$

$$\int \sqrt{x}\sqrt{1+x\sqrt{x}}\,dx = \frac{2}{3}\int u^{\frac{1}{2}}du = \frac{4}{9}u^{\frac{3}{2}} + c = \frac{4}{9}\left(1+x\sqrt{x}\right)^{\frac{3}{2}} + c$$

$50.-Resolver$: $\int x^3\,(x^2-3)^{1/2}dx$

$Solucion$: $u = x^2-3 \ \rightarrow x^2 = u+3 \ \rightarrow\ xdx = \frac{du}{2}$

$$\int x^3\,(x^2-3)^{\frac{1}{2}}dx = \int x^2(x^2-3)^{\frac{1}{2}}x\,dx = \int (u+3)u^{\frac{1}{2}}\frac{du}{2} = \frac{1}{2}\int (u+3)u^{\frac{1}{2}}\,du$$

$$= \frac{1}{2}\int (u^{\frac{3}{2}} + 3u^{\frac{1}{2}})dx$$

$$= \frac{1}{2}\int u^{\frac{3}{2}}\,du + \frac{3}{2}\int u^{\frac{1}{2}}\,du = \frac{2}{2}\cdot\frac{u^{\frac{5}{2}}}{5} + \frac{3}{2}\cdot\frac{2u^{\frac{3}{2}}}{3} - c = \frac{1}{5}u^{\frac{5}{2}} + u^{\frac{3}{2}} + c$$

$$\int x^3\,(x^2-3)^{\frac{1}{2}}\,dx = \frac{1}{5}(x^2-3)^{\frac{5}{2}} + (x^2-3)^{\frac{3}{2}} + c$$

$51.-Resolver\ \int \frac{xdx}{(ax+b)^{\frac{3}{2}}}$

$$Solución:\quad u = ax+b \quad\rightarrow\quad x = \frac{u-b}{a} \quad\rightarrow\quad dx = \frac{du}{a}$$

$$Resolver\ \int \frac{xdx}{(ax+b)^{\frac{3}{2}}} = \frac{1}{a^2}\int u^{-3/2}\,(u-b)du = \frac{1}{a^2}\int u^{-1/2}\,du - \frac{b}{a^2}\int u^{-3/2}du$$

$$\int \frac{xdx}{(ax+b)^{\frac{3}{2}}} = \frac{2}{a^2}u^{1/2} + \frac{2b}{a^2}u^{-1/2} + c = \frac{2}{a^2}\sqrt{ax+b} + \frac{2b}{a^2\sqrt{ax+b}} + c$$

$52.-Resolver\ \int \left(\frac{x^3+2}{x^3}\right)\sqrt{x-\frac{1}{x^2}}\,dx$

$$Solución:\ u = x - \frac{1}{x^2} \quad\rightarrow\quad du = \left(\frac{x^3+2}{x^3}\right)dx$$

$$\int \left(\frac{x^3+2}{x^3}\right)\sqrt{x-\frac{1}{x^2}}\,dx = \int u^{1/2}\,du = \frac{2}{3}u^{3/2} + c = \frac{2}{3}\left(x-\frac{1}{x^2}\right)^{3/2} + c$$

$53.-Resolver\!:\ \int \frac{dx}{\sec(2x)}$

$$Solución:\quad u = 2x \ \rightarrow\ \frac{du}{2} = dx$$

$$\int \frac{dx}{\sec(2x)} = \int \cos(2x)\,dx = \frac{1}{2}\int \cos u\,du = \frac{1}{2}\operatorname{sen} u + c = \frac{1}{2}\operatorname{sen}(2x) + c$$

$54.-Resolver\ \int \frac{\cot(3x)dx}{\operatorname{sen}^2(3x)}$

$Solución\!:\ u = 3x \quad \rightarrow \quad \frac{du}{3} = dx$

$$\int \frac{\cot(3x)dx}{\operatorname{sen}^2(3x)} = \int \csc^2(3x)\cot(3x)dx = \frac{1}{3}\int \cot u \csc^2 u\, du$$

$$m = \cot u \quad \rightarrow \quad -dm = \csc^2 u\, du$$

$$\frac{1}{3}\int \csc^2 u \cot u\, du = -\frac{1}{3}\int mdm = -\frac{1}{3}.\frac{1}{2}m^2 + c = -\frac{1}{6}m^2 + c = -\frac{1}{6}(\cot u)^2 + c$$
$$= -\frac{1}{6}(\cot(3x))^2 + c$$

$55.-Resolver\ \int x^2\cot(x^3)\csc(x^3)\, dx$

$Solución\!:\ u = x^3 \quad \rightarrow \quad \frac{du}{3} = x^2dx$

$$\int x^2\cot(x^3)\csc(x^3)\, dx = \frac{1}{3}\int \csc(u)\cot(u)du = \frac{1}{3}(-\csc u) + c = -\frac{1}{3}\csc x^3 + c$$

$56.-Resolver\ \int \sqrt{\tan(x)}\ \sec^2(x)\, dx$

$Solución\!:\ u = \tan(x) \quad \rightarrow \quad du = \sec^2(x)dx$

$$\int \sqrt{\tan(x)}\sec^2(x)\, dx = \int (\tan x)^{\frac{1}{2}} \sec^2 x\, dx + c$$

$$\int u^{\frac{1}{2}}\, du = \frac{2}{3}u^{\frac{3}{2}} + c = \frac{2}{3}(\tan x)^{\frac{3}{2}} + c$$

$57.-Resolver\ \int \sec(x)\big(\sec(x) - \tan(x)\big)dx$

$Solución\!:\ \int \sec x\,(\sec x - \tan x)dx = \int [\sec^2 x - (\sec x \tan x)]\, dx$

$$\int \sec^2 x\, dx - \int \sec x \tan x\, dx = \tan x - \sec x + c$$

$58.-Resolver \int (\operatorname{sen}(x))^{\frac{2}{3}} \cos(x)\, dx$

$Solución: u = \operatorname{sen}(x) \rightarrow du = \cos(x)\, dx$

$$\int (\operatorname{sen}(x))^{2/3} \cos(x)\, dx = \int u^{2/3}\, du = \frac{3}{5} u^{5/3} + c = \frac{3}{5} (\operatorname{sen} x)^{5/3} + c$$

$59.- Resolver \int \frac{\tan x\, dx}{\cos^2 x}$

$Solución: u = \tan x \rightarrow du = \sec^2 x\, dx$

$$\int \frac{\tan x\, dx}{\cos^2 x} = \int \frac{\tan x}{\frac{1}{\sec^2 x}} = \int \tan(x) \sec^2(x) dx = \int u\, du = \frac{1}{2} u^2 + c = \frac{1}{2} \tan^2 x + c$$

$60.-Resolver \int \frac{\operatorname{sen}(2x) dx}{\sqrt{1 + \operatorname{sen}^2 x}}$

$Solución: u = 1 + \operatorname{sen}^2 x \rightarrow du = 2 \operatorname{sen} x \cos x\, dx \rightarrow du = \operatorname{sen}(2x) dx$

$$\int \frac{\operatorname{sen}(2x) dx}{\sqrt{1 + \operatorname{sen}^2 x}} = \int \frac{2 \operatorname{sen} x \cos x\, dx}{\sqrt{1 + \operatorname{sen}^2 x}} = \int \frac{du}{u^{1/2}} = 2u^{1/2} + c = 2(1 + \operatorname{sen}^2 x)^{1/2} + c$$

$61.-Resolver \int \frac{dx}{\cos^2 x \sqrt{\tan(x) - 1}}$

$Solución: u = \tan x - 1 \rightarrow du = \sec^2 x\, dx$

$$\int \frac{dx}{\cos^2 x \sqrt{\tan(x) - 1}} = \int \frac{\sec^2 x\, dx}{\sqrt{\tan x - 1}} = \int \frac{\sec^2 x\, dx}{(\tan x - 1)^{1/2}} =$$

$$\int \frac{du}{u^{\frac{1}{2}}} = \int u^{-1/2}\, du = 2u^{1/2} + c = 2\sqrt{\tan x - 1} + c$$

$62.-Resolver \int \frac{\csc^2(\sqrt{x}) dx}{\sqrt{x}}$

$Solución: u = \sqrt{x} \rightarrow 2du = \frac{dx}{\sqrt{x}}$

$$\int \frac{\csc^2(\sqrt{x}) dx}{\sqrt{x}} = 2 \int \csc^2 u du = -2 \cot u + c = -2 \cot \sqrt{x} + c$$

63. $-Resolver\quad \int \frac{x^2.\operatorname{sen}(x^3+1)dx}{\cos^3(x^3+1)}$

$Solución:\ u = x^3+1 \quad \rightarrow \quad \frac{du}{3} = x^2dx$

$$\int \frac{x^2.\operatorname{sen}(x^3+1)dx}{\cos^3(x^3+1)} = \frac{1}{3}\int \frac{\operatorname{sen} u\, du}{\cos^3 u} = \frac{1}{3}\int \cos^{-3} u(\operatorname{sen} x\, dx)$$

$$m = \cos u \quad \rightarrow -dm = \operatorname{sen} x dx$$

$$\frac{1}{3}\int \cos^{-3} u(\operatorname{sen} x\, dx) = \frac{1}{3}\int m^{-3}(-dm) = -\frac{1}{3}\int m^{-3}dm = \frac{1}{6}m^{-2} + c$$

$$\int \frac{x^2.\operatorname{sen}(x^3+1)dx}{\cos^3(x^3+1)} = \frac{1}{6}\cos^{-2}u + c = \frac{1}{6}\cos^{-2}(x^3+1) + c$$

64. $-Resolver\quad \int \frac{dx}{\operatorname{sen}^2(ax+b)}$

$Solución:\ u = ax+b \quad \rightarrow \quad \frac{du}{a} = dx$

$$\int \frac{dx}{\operatorname{sen}^2(ax+b)} = \int \csc^2(ax+b)dx = \frac{1}{a}\int \csc^2 u\, du = -\frac{1}{a}\cot u + c = -\frac{1}{a}\cot(ax+b) + c$$

65. $-Resolver\quad \int x.\operatorname{sen}\left(\frac{x^2}{2}\right)dx$

$Solución:\quad u = \frac{x^2}{2} \quad \rightarrow \quad du = xdx$

$$\int x.\operatorname{sen}\left(\frac{x^2}{2}\right)dx = \int \operatorname{sen} u\, du = -\cos u + c = -\cos\left(\frac{x^2}{2}\right) + c$$

66. $-Resolver\quad \int \frac{\operatorname{sen}\sqrt{x}.\cos\sqrt{x}\, du}{\sqrt{x}}$

$Solución\quad u = \sqrt{x} \quad \rightarrow \quad 2du = \frac{dx}{\sqrt{x}}$

$Solución\ I$: $\int \frac{\operatorname{sen}\sqrt{x}.\cos\sqrt{x}du}{\sqrt{x}} = 2\int \operatorname{sen} u.\cos u\, du = \operatorname{sen}^2 u + c = \operatorname{sen}^2(\sqrt{x}) + c$

$Solución\ \ II$: $\displaystyle\int \frac{\operatorname{sen}\sqrt{x}.\cos\sqrt{x}\, du}{\sqrt{x}} = \int 2 \operatorname{sen} u.\cos u\ du$

$$\int \operatorname{sen}(2u)du = -\frac{1}{2}\cos(2u) + c = -\frac{1}{2}\cos\left(2\sqrt{x}\right) + c$$

$67. -Resolver$ $\displaystyle\int \sec^3 x \tan x\ dx$

$Solución$: $\displaystyle\int \sec^3 x \tan x\ dx = \int \sec^2 x(\sec x.\tan x)dx$

$$sea \quad u = \sec x\ , \quad du = \sec x \tan x\ dx$$

$$\int \sec^2 x(\sec x.\tan x)dx = \int u^2 du = \frac{1}{3}u^3 + C$$

$$porlo\ tanto \quad \int \sec^3 x \tan x\ dx = \frac{1}{3}\sec^3 x + c$$

INTEGRAL QUE CONDUCE A FUNCION TRIGONOMETRICA INVERSA

En los problemas que siguen a continuación se utilizan las siguientes fórmulas de integración:

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + c$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \text{arcsen}\left(\frac{u}{a}\right) + c\ ,$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\text{arcsec}\left(\frac{u}{a}\right) + c$$

1. −*Resolver*: $\int \frac{dx}{\sqrt{2-5x^2}}$

$$\int \frac{dx}{\sqrt{2-5x^2}} = \int \frac{dx}{\sqrt{(\sqrt{2})^2-(\sqrt{5}x)^2}}$$

Solución: $a=\sqrt{2}$, $u=\sqrt{5}x \quad \to \quad \frac{1}{\sqrt{5}}du = dx$

Entonces $\int \frac{dx}{\sqrt{2-5x^2}} = \frac{1}{\sqrt{5}}\int \frac{du}{\sqrt{a^2-u^2}} = \frac{1}{\sqrt{5}}\text{arcsen}\left(\frac{u}{a}\right) + c$

$$\int \frac{dx}{\sqrt{2-5x^2}} = \frac{1}{\sqrt{5}}\text{arcsen}\left(\frac{\sqrt{5}x}{\sqrt{2}}\right) + c$$

2. −*Resolver*: $\int \frac{dx}{\sqrt{2+x-x^2}}$

Solucion: $\int \frac{dx}{\sqrt{2+x-x^2}} = \int \frac{dx}{\sqrt{2+\frac{1}{4}-(x^2-x+\frac{1}{4})}} = \int \frac{dx}{\sqrt{\frac{9}{4}-(x-\frac{1}{2})^2}}$

$$a=\frac{3}{2}\ ,\quad u = x-\frac{1}{2}\ ,\quad du = dx$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \text{arcsen}\left(\frac{u}{a}\right) + c \quad \Rightarrow \text{arcsen}\left(\frac{x-\frac{1}{2}}{\frac{3}{2}}\right) + c = \text{arcsen}\left(\frac{2x-1}{3}\right) + c$$

$3.-Resolver$: $\int \frac{dx}{x^2+2x+10}$

$Solucion$: $\int \frac{dx}{x^2+2x+10} = \int \frac{dx}{9+(x^2+2x+1)} = \int \frac{dx}{3^2+(x+1)^2} = \frac{1}{3}\arctan\left(\frac{x+1}{3}\right) + c$

$4.-Resolver$ $\int \frac{xdx}{x^4+3} = \int \frac{xdx}{(\sqrt{3})^2+(x^2)^2}$

$Solución$: $a=\sqrt{3}$, $u=x^2 \rightarrow \frac{du}{2} = xdx$

$Entonces$ $\int \frac{xdx}{(\sqrt{3})^2+(x^2)^2} = \frac{1}{2}\int \frac{du}{a^2+u^2} = \frac{1}{2}.\frac{1}{a}\arctan\left(\frac{u}{a}\right) + c$

$$\int \frac{xdx}{x^4+3} = \frac{1}{2\sqrt{3}}\arctan\left(\frac{x^2}{\sqrt{3}}\right) + c$$

$5.-Resolver$ $\int \frac{dx}{x\sqrt{4x^2-9}} = \int \frac{dx}{x\sqrt{(2x)^2-3^2}} = 2\int \frac{dx}{2x\sqrt{(2x)^2-3^2}}$

$Solución$: $a=3$, $u=2x \rightarrow \frac{du}{2} = dx$

$Entonces$ $2\int \frac{dx}{2x\sqrt{(2x)^2-3^2}} = \frac{2}{2}\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\left(\frac{u}{a}\right) + c$

$$\int \frac{dx}{x\sqrt{4x^2-9}} = \frac{1}{3}\operatorname{arcsec}\left(\frac{2x}{3}\right) + c$$

$6.-Resolver$ $\int \frac{x^2dx}{\sqrt{1-x^6}} = \int \frac{x^2dx}{\sqrt{1-(x^3)^2}}$

$Solución$: $a=1$, $u=x^3 \rightarrow \frac{du}{3} = x^2dx$

$Entonces$ $\int \frac{x^2dx}{\sqrt{1-(x^3)^2}} = \frac{1}{3}\int \frac{du}{\sqrt{a^2-u^2}} = \frac{1}{3}\operatorname{arcsen}\left(\frac{u}{a}\right) + c$

$$\int \frac{x^2dx}{\sqrt{1-x^6}} = \frac{1}{3}\operatorname{arcsen}(x^3) + c$$

$7.-\ Resolver\text{:} \quad \int \frac{xdx}{x^2\sqrt{x^4-1}}$

$Solución\text{:} \quad a = 1\ , \qquad u = x^2 \ \rightarrow\ \frac{du}{2} = xdx$

$$luego \quad \int \frac{xdx}{x^2\sqrt{x^4-1}} = \frac{1}{2}\int \frac{du}{u\sqrt{u^2-a^2}}$$

$$\frac{1}{2}.\frac{1}{a}\operatorname{arcsec}\left(\frac{x^2}{a}\right) + c = \frac{1}{2}\operatorname{arcsec}(x^2) + c$$

$8.-\ Resolver\text{:} \quad \int \frac{axdx}{x^4+b^4}$

$Solución\text{:}\ d = b^2\ , \quad u = x^2 \rightarrow\ \frac{du}{2} = xdx$

$$\int \frac{axdx}{x^4+b^4} = \frac{a}{2}\int \frac{du}{u^2+d^2} = \frac{a}{2}.\frac{1}{d}\arctan\left(\frac{u}{d}\right) + c = \frac{a}{2b^2}\arctan\left(\frac{x^2}{b^2}\right) + c$$

$9.-Resolver \quad \int \frac{(x+2)dx}{\sqrt{4-2x-x^2}}$

$Solución\text{:} \quad \int \frac{(x+2)dx}{\sqrt{4-2x-x^2}} = \int \frac{(x+2)dx}{\sqrt{(4+1)-(x^2+2x+1)}} = \int \frac{(x+2)dx}{\sqrt{5-(x+1)^2}}$

$$= \int \frac{[(x+1)+1]dx}{\sqrt{5-(x+1)^2}}$$

$$\int \frac{(x+2)dx}{\sqrt{5-(x+1)^2}} = \int \frac{(x+1)dx}{\sqrt{5-(x+1)^2}} + \int \frac{dx}{\sqrt{5-(x+1)^2}}$$

$$I = \int \frac{(x+1)dx}{\sqrt{4-2x-x^2}},$$

$$hacemos \quad u = 4-2x-x^2, \qquad du = -2(1+x)dx, \qquad -\frac{du}{2} = (x+1)dx$$

$$\int \frac{(x+1)dx}{\sqrt{4-2x-x^2}} = -\frac{1}{2}\int \frac{du}{u^{1/2}}$$

$$-\frac{1}{2}\int u^{-1/2}du = (-\frac{1}{2})\ \frac{u^{1/2}}{\frac{1}{2}} = -u^{1/2} = -(4-2x-x^2)^{1/2}$$

$$II = \int \frac{dx}{\sqrt{5-(x+1)^2}},$$

$$hacemos\ \ u = x+1, \qquad du = dx, \quad a = \sqrt{5}$$

$$\int \frac{dx}{\sqrt{5-(x+1)^2}} = \int \frac{du}{\sqrt{a^2-u^2}} = \operatorname{arcsen}\left(\frac{x+1}{\sqrt{5}}\right) + c$$

$$Luego\ \ \int \frac{(x+2)dx}{\sqrt{4-2x-x2}} = \ -(4-2x-x^2)^{1/2} + \operatorname{arcsen}\left(\frac{x+1}{\sqrt{5}}\right) + c$$

$10.-Resolver$: $\int \frac{dx}{x^2+10x+30}$

$$\int \frac{dx}{x^2+10x+30} = \int \frac{dx}{x^2+10x+25+5} = \int \frac{dx}{(x+5)^2+5}$$

$Solucion$: $a = \sqrt{5}$ $,u = x+5 \quad \rightarrow \quad \mathrm{du} = \mathrm{dx}$

$$Entonces \quad \int \frac{dx}{(x+5)^2+5} = \int \frac{du}{u^2+a^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + c = \frac{\sqrt{5}}{5}\arctan\left(\frac{x+5}{\sqrt{5}}\right) + c$$

$11.-Resolver$: $\int \frac{dx}{\sqrt{4-(x+2)^2}}$

$Solución$: $a = 2$, $\quad u = x+2 \quad \rightarrow \quad du = dx$

$$\int \frac{dx}{\sqrt{4-(x+2)^2}} = \int \frac{du}{\sqrt{a^2-u^2}} = \operatorname{arcsen}\left(\frac{u}{a}\right) + c = \operatorname{arcsen}\left(\frac{x+2}{2}\right) + c$$

$12.-Resolver$: $\int \frac{\sec(x)\tan(x)dx}{9-4\sec^2(x)}$

$Solución$: $a = 3$, $\quad u = 2\sec(x) \quad \rightarrow \quad \frac{du}{2} = \sec(x)\tan(x)dx$

$$\int \frac{\sec(x)\tan(x)dx}{9-4\sec^2(x)} = \frac{1}{2}\int \frac{du}{a^2-u^2} = \frac{1}{2}.\frac{1}{2a}.\ln\left(\frac{a+u}{a-u}\right) + c = \frac{1}{12}\ln\left(\frac{3+2\sec(x)}{3-2\sec(x)}\right) + c$$

$13.-Resolver$: $\int \frac{\cos(x)dx}{1+\text{sen}^2(x)}$

$Solución$: $a = 1$, $u = sen(x) \rightarrow du = cos(x)dx$

$$Entonces \quad \int \frac{\cos(x)dx}{1+\text{sen}^2(x)} = \int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + c =$$

$$\int \frac{\cos(x)dx}{1+sen^2(x)} = \arctan\big(\text{sen}(x)\big) + c$$

$14.-Resolver$: $\int \frac{dx}{\sqrt{20+8x-x^2}}$

$$Solucion: \int \frac{dx}{\sqrt{20+8x-x^2}} = \int \frac{dx}{\sqrt{20+16-(x^2-8x+16)}} = \int \frac{dx}{\sqrt{36-(x-4)^2}}$$

$si\ se\ hace \quad a = 6$, $u = x-4 \rightarrow du = dx$

$$Entonces \quad \int \frac{dx}{\sqrt{36-(x-4)^2}} = \int \frac{du}{\sqrt{a^2-u^2}} = \text{arcsen}\left(\frac{u}{a}\right) + c$$

$$\int \frac{dx}{\sqrt{36-(x-4)^2}} = \text{arcsen}\left(\frac{x-4}{6}\right) + c$$

$15.-Resolver$: $\int \frac{dx}{\sqrt{2x-x^2}}$

$$\int \frac{dx}{\sqrt{2x-x^2}} = \int \frac{dx}{\sqrt{1-(x^2-2x+1)}} = \int \frac{dx}{\sqrt{1-(x-1)^2}}$$

$Solución$: $a = 1,$ $u = x-1 \rightarrow du = dx$

$$\int \frac{dx}{\sqrt{1-(x-1)^2}} = \int \frac{dx}{\sqrt{a^2-u^2}} = \text{arcsen}\left(\frac{u}{a}\right) + c$$

$$\int \frac{dx}{\sqrt{2x-x^2}} = \text{arcsen}(x-1)$$

$16.-Resolver$: $\int \frac{dx}{\sqrt{x}(1+x)}$

$Solucion$: $a = 1$ $u = \sqrt{x} \rightarrow 2du = \frac{dx}{\sqrt{x}}$

$$\int \frac{dx}{\sqrt{x}(1+x)} = 2\int \frac{du}{a^2+u^2} = \frac{2}{a}\arctan\left(\frac{u}{a}\right) + c$$

$$\int \frac{dx}{\sqrt{x}(1+x)} = 2\text{arcot}(\sqrt{x}) + c$$

$17.-Resolver$: $\int \frac{dx}{x\sqrt{1-\ln^2(x)}}$

$Solucion$: $a = 1$, $u = \ln(x) \rightarrow du = \frac{dx}{x}$

$$\int \frac{dx}{x\sqrt{1-\ln^2(x)}} = \int \frac{du}{\sqrt{a^2-u^2}} = \text{arcsen}\left(\frac{u}{a}\right) + c$$

$$\int \frac{dx}{x\sqrt{1-\ln^2(x)}} = \text{arcsen}(\ln(x)) + c$$

$18.-Resolver$: $\int \frac{e^x dx}{\sqrt{1-e^{2x}}}$

$Solucion$: $a = 1$, $u = e^x \rightarrow du = e^x dx$, $\int \frac{e^x dx}{\sqrt{1-e^{2x}}} = \int \frac{du}{\sqrt{a^2-u^2}} = \text{arcsen}\left(\frac{u}{a}\right) + c$

$$\int \frac{e^x dx}{\sqrt{1-e^{2x}}} = \text{arcsen}(e^x) + c$$

$19.-Resolver$: $\int \frac{x^2 dx}{x^6+25}$

$$\int \frac{x^2 dx}{x^6+25} = \int \frac{x^2 dx}{5^2+(x^3)^2}$$

$Solucion$: $a = 5$, $u = x^3 \rightarrow \frac{du}{3} = x^2 dx$

$$\int \frac{x^2 dx}{5^2+(x^3)^2} = \frac{1}{3}\int \frac{du}{a^2+u^2} = \frac{1}{3}.\frac{1}{a}\text{arctan}\frac{u}{a} + c = \frac{1}{15}\text{arcot}\frac{x^3}{5} + c$$

$20.-Resolver$ $\int \frac{\cos(x)\text{sen}(x)dx}{1+\text{sen}^4(x)} = \int \frac{\cos(x)\text{sen}(x)dx}{1+(\text{sen}^2(x))^2}$

$Solución$: $a = 1$ $u = \text{sen}^2 x$ $\frac{du}{2} = \text{sen}(x)\cos(x)dx$

$$Entonces \quad \int \frac{\cos(x)\text{sen}(x)dx}{1+[\text{sen}^2(x)]^2} = \frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}.\frac{1}{a}\text{arcot}\left(\frac{u}{a}\right) + c$$

$$\int \frac{\cos(x)\text{sen}(x)dx}{1+\text{sen}^4(x)} = \frac{1}{2}\text{arctan}(\text{sen}^2(x)) + c$$

$$21.-Resolver \int \frac{dx}{4x^2-4x+2} = \int \frac{dx}{[(2x)^2-2(2x)+1]+1} = \int \frac{dx}{(2x-1)^2+1}$$

$Solución$: $a = 1$ $u = 2x - 1$ $\frac{du}{2} = dx$

$$Entonces \quad \int \frac{dx}{4x^2 4x+2} = \frac{1}{2}\int \frac{du}{u^2+a^2} = \frac{1}{2}.\frac{1}{a}\arctan\left(\frac{u}{a}\right)+c$$

$$\int \frac{dx}{4x^2-4x+2} = \frac{1}{2}\arctan(2x-1)+c$$

$$22.-Resolver \int \frac{2\,\mathrm{sen}(4x)dx}{1+\mathrm{sen}^4(2x)}$$

$$\int \frac{2\,\mathrm{sen}(4x)dx}{1+\mathrm{sen}^4(2x)} = \int \frac{2\mathrm{sen}(4x)dx}{1+(\mathrm{sen}^2(2x))^2}$$

$Solución$: $a = 1$, $u = \mathrm{sen}^2(2x)$ $du = 2\,\mathrm{sen}(2x)\cos(2x)(2)dx$

$$4\,\mathrm{sen}(2x)\cos(2x)dx = 2\mathrm{sen}(4x)dx$$

$$Entonces \quad \int \frac{2\,\mathrm{sen}(4x)dx}{1+\mathrm{sen}^4(2x)} = \int \frac{du}{a^2+u^2} = \arctan\left(\frac{u}{a}\right)+c$$

$$\int \frac{2\,\mathrm{sen}(4x)dx}{1+\mathrm{sen}^4(2x)} = \arctan\left(\mathrm{sen}^2(2x)\right)+c$$

$$23.-Resolver \int \frac{\csc^2(3x)dx}{25+\cot^2(3x)}$$

$Solución$: $a = 5$, $u = \cot(3x)$, $\frac{du}{3} = -\csc^2(3x)dx$

$$Entonces \quad \int \frac{\csc^2(3x)dx}{25+\cot^2(3x)} = -\frac{1}{3}\int \frac{du}{a^2+u^2}$$

$$-\frac{1}{3.a}\arctan\left(\frac{u}{a}\right) = -\frac{1}{15}\arctan\left(\frac{u}{a}\right)+c$$

$$\int \frac{\csc^2(3x)dx}{25+\cot^2(3x)} = -\frac{1}{15}\arctan\left(\frac{\cot(3x)}{5}\right)+c$$

$24.-Resolver \int \frac{dx}{\sqrt{12-4x-x^2}}$

$$\int \frac{dx}{\sqrt{12-4x-x^2}} = \int \frac{dx}{\sqrt{(12+4)-(x^2+4x+4)}} = \int \frac{dx}{\sqrt{16-(x+2)^2}}$$

$Solución: \quad a = 4 \qquad u = x+2 \qquad du = dx$

$$\int \frac{dx}{\sqrt{12-4x-x^2}} = \int \frac{du}{\sqrt{a^2-u^2}} = \operatorname{arcsen}\left(\frac{u}{a}\right) + c$$

$$\int \frac{dx}{\sqrt{12-4x-x^2}} = \operatorname{arcsen}\left(\frac{x+2}{4}\right) + c$$

$25.-Resolver \int \frac{dx}{\sqrt{1-a^2x^2}}$

$$\int \frac{dx}{\sqrt{1-a^2x^2}} = \int \frac{dx}{\sqrt{1-(ax)^2}}$$

$Solución: \quad u = \ ax \qquad \frac{du}{a} = dx$

$$Entonces \quad \int \frac{dx}{\sqrt{1-(ax)^2}} = \frac{1}{a}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{a}\operatorname{arcsen}(ax) + c$$

$$\int \frac{dx}{\sqrt{1-(ax)^2}} = \frac{1}{a}\operatorname{arcsen}(ax) + c$$

$26.-Resolver \int \frac{dx}{\sqrt{9-4x^2}}$

$$Solución: \int \frac{dx}{\sqrt{9-4x^2}} = \int \frac{dx}{\sqrt{3^2-(2x)^2}}$$

$a = 3 \qquad u = 2x \qquad \frac{du}{2} = dx$

$$Entonces \quad \int \frac{dx}{\sqrt{3^2-(2x)^2}} = \frac{\mathbf{1}}{\mathbf{2}}\int \frac{du}{\sqrt{a^2-u^2}} = \frac{1}{2}\operatorname{arcsen}\left(\frac{u}{a}\right) + c$$

$$\int \frac{dx}{\sqrt{9-4x^2}} = \frac{1}{2}\operatorname{arcsen}\left(\frac{2x}{3}\right) + c$$

INTEGRAL QUE CONDUCE A EXPRESION

LOGARITMICAO EXPONENCIAL

$1.-Resolver \int \frac{xdx}{1+x^2}$

$Solución:\ u = x^2 + 1 \quad \rightarrow \quad \frac{du}{2} = xdx$

$$\int \frac{xdx}{1+x^2} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln u + c = \frac{1}{2}\ln(x^2+1) + c$$

$2.-Resolver \int xe^{x^2}dx$

$Solución:\ u = x^2 \quad \rightarrow \quad \frac{du}{2} = \quad xdx$

$$\int xe^{x^2}dx = \int e^{x^2}\,xdx = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + c = \frac{1}{2}e^{x^2} + c$$

$3.-Resolver \int \frac{dx}{x\ln x}$

$Solución\quad u = lnx \quad \rightarrow \quad du = \frac{dx}{x},\ \ luego\quad \int \frac{dx}{x\ln x} = \int \frac{du}{u} = \ln u = \ln(\ln x) + c$

$4.-Resolver \int \frac{\ln(x)dx}{x}$

$Solución:\ u = \ln(x) \quad \rightarrow \quad du = \frac{dx}{x}$

$$\int \frac{\ln(x)dx}{x} = \int u\,du = \frac{1}{2}u^2 + c = \frac{1}{2}(\ln x)^2 = \frac{1}{2}\ln^2(x) + c$$

$5.-Resolver \int \frac{cos\big(\ln(x)\big)dx}{x}$

$Solución:\ u = \ln x,\ \ du = \frac{dx}{x},\quad \int \frac{\cos\big(\ln(x)\big)dx}{x} = \int \cos u\,du = \mathrm{sen}\,u + c = \mathrm{sen}(\ln x) + c$

$6.-Resolver \int \frac{\text{sen}(x)dx}{\cos(x)}$

$$Solución: \quad u = \cos x \quad \rightarrow \quad -du = \text{sen}\, x\, dx$$

$$\int \frac{\text{sen}(x)dx}{\cos(x)} = -\int \frac{du}{u} = -\ln u + c = -\ln(\cos x) + c$$

$7.-Resolver \int \tan(x)dx$

$Solución: \quad u = \cos(x) \quad \rightarrow \quad -du = \text{sen}(x)dx$

$$\int \tan(x)dx = \int \frac{\text{sen}(x)}{\cos(x)}dx = -\int \frac{du}{u} = \ln(u) + c = -\ln\big(\cos(x)\big) + c$$

$8.-Resolver \int \frac{(e^x + 1)dx}{e^x}$

$Solución: \int \frac{(e^x + 1)dx}{e^x} = \int \left(\frac{e^x}{e^x} + \frac{1}{e^x}\right)dx$

$$\int dx + \int e^{-x}\, dx = x - e^{-x} + c$$

$9.-Resolver \int \frac{e^x dx}{(e^x - 5)}$

$$Solución: \quad u = e^x - 5 \quad \rightarrow \quad du = e^x dx$$

$$\int \frac{e^x dx}{(e^x - 5)} = \int \frac{du}{u} = \ln u + c = \ln(e^x - 5) + c$$

$10.-Resolver \int \sec^2(x)e^{\tan(x)}dx$

$Solución: \quad u = \tan x \quad \rightarrow \quad du = \sec^2 x\, dx$

$$\int \sec^2(x)e^{\tan(x)}dx = \int e^u\, du = e^u + c =$$

$$\int \sec^2(x)e^{\tan(x)}dx = \ e^{\tan x} + c$$

$11.-Resolver \int \frac{(e^x+\sec^2 x)dx}{(\tan x + e^x)}$

$Solución$: $u = \tan x + e^x \quad \rightarrow \quad du = (\sec^2 x + e^x)dx$

$$\int \frac{(e^x+\sec^2 x)dx}{(\tan x + e^x)} = \int \frac{du}{u} = \ln u + c = \ln(\tan x + e^x) + c$$

$12.-Resolver \int \frac{(e^{2x}+e^x+1)dx}{e^x}$

$Solución$: $\int \frac{(e^{2x}+e^x+1)dx}{e^x} = \int \frac{e^{2x}}{e^x}dx + \int \frac{e^x}{e^x}dx + \int \frac{dx}{e^x}$

$$\int e^x\,dx + \int dx + \int e^{-x}dx = e^x + x - e^{-x} + c$$

$13.-Resolver \int \frac{\sqrt{x}dx}{1+x\sqrt{x}}$

$Solución$: $u = 1 + x\sqrt{x} \quad \rightarrow \quad du = \frac{3\sqrt{x}}{2}dx \quad \rightarrow \quad \frac{2}{3}du = \sqrt{x}dx$

$$\int \frac{\sqrt{x}dx}{1+x\sqrt{x}} = \frac{2}{3}\int \frac{du}{u} + c = \frac{2}{3}\ln u + c$$

$$\int \frac{\sqrt{x}dx}{1+x\sqrt{x}} = \frac{2}{3}\ln(1+x\sqrt{x}) + c$$

$14.-Resolver \int \frac{e^x dx}{4-e^x}$

$Solución$; $u = -e^x + 4 \quad \rightarrow \quad -du = e^x dx$

$$\int \frac{e^x dx}{4-e^x} = -\int \frac{du}{u} = -\ln u + c$$

$$\int \frac{e^x dx}{4-e^x} = -\ln(4-e^x) + c$$

$15.-Resolver \int \frac{e^{\sqrt{x+1}}dx}{\sqrt{x+1}}$

$Solución$: $u = \sqrt{x+1} \quad \rightarrow \quad 2\,du = \frac{dx}{\sqrt{x+1}}$

$$\int \frac{e^{\sqrt{x+1}}dx}{\sqrt{x+1}} = 2\int e^u\,du = 2\int e^u\,du = 2e^{\sqrt{x+1}} + c$$

$16.-Resolver \;\; \int \frac{(e^x + e^{-x})dx}{(e^x - e^{-x})}$

$Solución$: $u = e^x - e^{-x} \quad \rightarrow \quad du = (e^x + e^{-x})dx$

$$\int \frac{(e^x + e^{-x})dx}{(e^x - e^{-x})} = \int \frac{du}{u} = \ln u + c = \ln(e^x - e^{-x}) + c$$

$17.-Resolver \;\; \int \frac{dx}{x\,\ln\sqrt{x}}$

$Solución$: $u = \ln\sqrt{x} \quad \rightarrow \; 2\,du = \frac{dx}{x}$

$$\int \frac{dx}{x\,\ln\sqrt{x}} = 2\int \frac{du}{u} = 2\ln u + c = 2\ln\left(\ln\sqrt{x}\right) + c$$

$18.-Resolver \;\; \int \frac{dx}{e^x + 1}$

$Solución$: $\int \frac{dx}{e^x + 1} = \int \frac{e^{-x}dx}{e^{-x}(e^x + 1)} + c \; = \int \frac{e^{-x}dx}{(1 + e^{-x})} + \; c$

$$si \;\; u = 1 + e^{-x} \quad entonces \quad -du = e^{-x}dx$$

$$luego \;\; \int \frac{e^{-x}dx}{(1 + e^{-x})} + \; c \quad = -\int \frac{du}{u} = -\ln(u) + c$$

$$\int \frac{dx}{e^x + 1} = -\ln(1 + e^{-x}) + c$$

$19.-Resolver \;\; \int \frac{\left(e^{\sqrt{x}} + 1\right)dx}{\sqrt{x}}$

$Solución$: $u = \sqrt{x} \quad \rightarrow \quad 2du = \frac{dx}{\sqrt{x}}$

$$\int \frac{\left(e^{\sqrt{x}} + 1\right)dx}{\sqrt{x}} = \int \frac{e^{\sqrt{x}}dx}{\sqrt{x}} + \int \frac{dx}{\sqrt{x}} = 2\int e^u\,du + 2\int du = 2e^u + 2u + c$$

$$\int \frac{\left(e^{\sqrt{x}}+1\right)dx}{\sqrt{x}} = 2e^{\sqrt{x}} + 2x^{1/2} + c = 2e^{\sqrt{x}} + 2\sqrt{x} + c$$

$20.-Resolver\ \int \frac{\ln(x)dx}{x(1+\ln(x))^{1/2}}$

$Solución\colon\ u = 1+\ln(x) \quad \rightarrow \quad du = \frac{dx}{x} \ , \quad u-1 = \ln(x)$

$$\int \frac{\ln(x)dx}{x(1+\ln(x))^{1/2}} = \int \frac{(u-1)du}{u^{1/2}} = \int \frac{u\,du}{u^{1/2}} - \int \frac{du}{u^{1/2}}$$

$$\int u^{1/2}\,du - \int u^{-1/2} = \frac{2\,u^{3/2}}{3} - 2u^{1/2} + c$$

$$\int \frac{\ln(x)dx}{x(1+\ln(x))^{1/2}} = \frac{2}{3}[1+\ln(x)]^{3/2} - 2[1+\ln(x)]^{1/2} + c$$

$21.-Resolver \int e^{3x}.e^{2x}dx$

$Solución\colon\ \int e^{3x}.e^{2x}dx = \int e^{5x}\,dx = \frac{1}{5}e^{5x} + c$

$22.-Resolver\ \int \frac{e^{-x}dx}{e^{-x}+1}$

$Solución\colon\ u = e^{-x}+1 \quad \rightarrow \ -du = e^{-x}dx$

$$\int \frac{e^{-x}dx}{e^{-x}+1} = -\int \frac{du}{u} = -\ln u + c = -\ln(e^{-x}+1) + c$$

$23.-Resolver\ \int (2x+1)10^{(3x^2+3x+1)}dx$

$Solución\colon\ u = 3x^2+3x+1 \quad \rightarrow \ \frac{du}{3} = (2x+1)dx$

$$\int (2x+1)10^{(3x^2+3x+1)}\,dx = \frac{1}{3}\int 10^u\,du + c = \frac{1}{3}\frac{10^u}{\ln 10} + c$$

$$\int (2x+1)10^{(3x^2+3x+1)} = \frac{10^{(3x^2+3x+1)}}{3\ln 10} + c$$

$24.-Resolver\ \int (e^{-3x} + \frac{1}{e^x})dx$

Solución:

$$\int (e^{-3x} + \frac{1}{e^x})dx = \int e^{-3x}\,dx + \int \frac{dx}{e^x} = \int e^{-3x}\,dx + \int e^{-x}\,dx = -\frac{1}{3}e^{-3x} - e^{-x} + c$$

$25.-\ Resolver\ \int \frac{e^{3x}dx}{(1-2e^{3x})^2}$

$$Solución\!:\ u = 1 - 2e^{3x} \quad \rightarrow \quad -\frac{du}{6} = e^{3x}dx$$

$$\int \frac{e^{3x}dx}{(1-2e^{3x})^2} = -\frac{1}{6}\int \frac{du}{u^2} = -\frac{1}{6}\int u^{-2}\,du = \frac{1}{6}u^{-1} + c = \frac{1}{6(1-2e^{3x})}$$

$26.-Resolver\ \int a^{x\ln(x)}\big(1+\ln(x)\big)dx$

$$Solución\!:\ u = x\ln x \quad \rightarrow\ du = (\ln(x)+1)dx$$

$$\int a^{x\ln(x)}\big(1+\ln(x)\big)dx$$

$$\int a^{x\ln x(x)}\big(1+\ln(x)\big)dx = \int a^u\,.\,du = \frac{a^u}{\ln(a)} + c = \frac{a^{x\ln x}}{\ln(a)} + \boldsymbol{c}$$

$27.-Resolver\ \int \frac{e^{2x}dx}{e^{4x}+3}$

$$Solución\!:\ a = \sqrt{3}\quad ,\quad u = e^{2x} \quad \rightarrow \quad \frac{du}{2} = e^{2x}dx$$

$$\int \frac{e^{2x}dx}{e^{4x}+3} = \frac{1}{2}\int \frac{du}{u^2+a^2} = \frac{1}{2}.\frac{1}{a}\operatorname{arcot}\frac{u}{a} + c = \frac{1}{2\sqrt{3}}\operatorname{arcot}\frac{e^{2x}}{\sqrt{3}} + c$$

$28.-Resolver\ \int \frac{(2+(\ln x)^2)dx}{x\big(1-\ln(x)\big)}$

$$Solución\!:\ u = 1-\ln(x) \quad \Rightarrow -\ du = \frac{dx}{x} \quad ,\quad \ln(x) = 1-u \quad \Rightarrow\ \ln^2(x) = (1-u)^2$$

$$\int \frac{(2+(\ln x)^2)dx}{x\big(1-\ln(x)\big)} = -\int \frac{[2+(1-u)^2]du}{u} + c = -\int \frac{[2+1-2u+u^2]}{u} + c$$

$$-\int \frac{(3-2u+u^2)du}{u} + c = -3\int \frac{du}{u} + 2\int du \; - \int udu + c$$

$$\int \frac{(2+(\ln(x))^2 dx}{x\big(1-\ln(x)\big)} = \; -3\ln(u) + 2u \; + \frac{1}{2}u^2 + c$$

$$\int \frac{(2+\ln^2(x))dx}{x\big(1-\ln(x)\big)} \; = -3\ln\big(1-\ln(x)\big) + 2\big(1-\ln(x)\big) - \frac{1}{2}\big(1-\ln(x)\big)^2 \; + c$$

$29.-Resolver \; \int (e^{\frac{x}{2}} - e^{\frac{-x}{3}})dx$

$Solución\text{:} \;\; u = \frac{x}{2} \;\; \Rightarrow 2du = dx \;\;\; , \;\; v = -\frac{x}{3} \;\; \Rightarrow \; -3dv = dx$

$$\int (e^{\frac{x}{2}} - e^{-x/3})dx = \int e^{x/2}dx - \int e^{-x/3}\,dx + c = 2\int e^u du \; + 3\int e^v\,dv + c = \; 2e^u + 3e^v + c$$

$$\int (e^{\frac{x}{2}} - e^{-x/3})dx = 2e^{\frac{x}{2}} + 3e^{-\frac{x}{3}} + c$$

$30.-Resolver \; \int (x-1)e^{-x^2+2x}dx$

$Solución\text{:} \;\; u = -x^2 + 2x \;\;\; \rightarrow \;\; -\frac{du}{2} = (x-1)dx$

$$\int (x-1)e^{-x^2+2x}dx = -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u + c = -\frac{1}{2}e^{-x^2+2x} + c$$

$31.-Resolver \; \int \frac{dx}{e^x + e^{-x}}$

$$\int \frac{dx}{e^x + e^{-x}} = \int \frac{dx}{\frac{e^x}{e^x}(e^x + e^{-x})} = \int \frac{dx}{e^x(1+e^{-2x})} = \int \frac{e^{-x}dx}{1+e^{-2x}}$$

$Solución\text{:} \;\; u = e^{-x} \;\;\;\; \rightarrow -du = e^{-x}dx$

$$\int \frac{e^{-x}dx}{1+e^{-2x}} = \int \frac{-du}{1+u^2} = -\int \frac{du}{1+u^2} = -\mathrm{arcot}(u) + c = -\mathrm{arcot}(e^{-x}) + c$$

$32.-Resolver \int (e^{x/2} - e^{-x/2})dx$

$$\int (e^{x/2} - e^{-x/2})dx = \int e^{x/2}\,dx - \int e^{-x/2}dx = 2e^{x/2} + 2e^{-x/2} + c$$

$$\int (e^{x/2} - e^{-x/2})dx = 2\left(e^{x/2} + e^{-x/2}\right) + c$$

$33.-Resolver \int \frac{dx}{\sqrt{e^x}}$

$Solución:\ \int \frac{dx}{\sqrt{e^x}} = \int e^{-x/2}dx = -\,2e^{-x/2} + c$

$34.-Resolver \int \frac{dx}{3-x}$

$Solución:\ u = 3 - x \quad \rightarrow \quad -du = dx$

$$\int \frac{dx}{3-x} = -\int \frac{du}{u} = -\ln(u) + c = -\ln(\,3 - x) + c$$

$35.-Resolver \int \frac{dx}{x(\ln x)^4}$

$Solución:\ u = \ln(x) \quad \rightarrow \quad du = \frac{dx}{x}$

$$\int \frac{dx}{x(\ln x)^4} = \int \frac{du}{u^4} = \int u^{-4}\,du = \frac{u^{-3}}{-3} + c = -\frac{1}{3.\ln^3(x)} + c$$

$36.-Resolver \int \frac{e^{2x}dx}{\sqrt{e^{4x}+1}}$

$Solución:\ a = 1\ ,\ \ u = e^{2x} \quad \rightarrow \quad \frac{du}{2} = e^{2x}dx$

$$\int \frac{e^{2x}dx}{\sqrt{e^{4x}+1}} = \frac{1}{2}\int \frac{du}{\sqrt{u^2+a^2}} = \frac{1}{2}\ln(u + \sqrt{u^2+a^2}) + c$$

$$\int \frac{e^{2x}dx}{\sqrt{e^{4x}+1}} = \ \frac{1}{2}\ln(e^{2x} + \sqrt{e^{4x}+1}) + c$$

$37.-Resolver\ \int \frac{e^{x^{-2}}}{x^3}dx$

$Solución:\ u = x^{-2} \quad \rightarrow -\frac{du}{2} = x^{-3}dx$

$$\int \frac{e^{x^{-2}}}{x^3}dx = -\frac{1}{2}\int e^u du = -\frac{1}{2}e^u + c = -\frac{1}{2}e^{x^{-2}} + c$$

$38.-Resolver \int e^{(-x^2+2)}xdx$

$Solución:\ u = -x^2 + 2 \quad \rightarrow -\frac{du}{2} = xdx$

$$\int e^{(-x^2+2)}xdx = \int e^u.(-\frac{du}{2}) = -\frac{1}{2}\int e^u du = -\frac{1}{2}e^u + c = -\frac{1}{2}e^{(-x^2+2)} + c$$

$39.-Resolver\ \int x^2 e^{x^3}dx$

$Solución:\ u = x^3 \quad \rightarrow \quad \frac{du}{3} = x^2dx$

$$\int x^2 e^{x^3}dx = \frac{1}{3}\int e^u.du = \frac{1}{3}e^u + c = \frac{1}{3}e^{x^3} + c$$

$40.-Resolver\ \int e^{\operatorname{sen}^2(x)}.\operatorname{sen}(2x)dx$

$Solución:\ u = \operatorname{sen}^2(x) \quad \rightarrow \quad du = 2\operatorname{sen}(x)\cos(x)dx = \operatorname{sen}(2x)dx$

$$\int e^{\operatorname{sen}^2(x)}.\operatorname{sen}(2x)dx = \int e^{\operatorname{sen}^2(x)}.2\operatorname{sen}(x)\cos(x)dx$$

$$\int e^{\operatorname{sen}^2(x)}.2\operatorname{sen}(x)\cos(x)dx = \int e^u.du = e^u + c = e^{\operatorname{sen}^2(x)} + c$$

$41.-Resolver\ \int \frac{\sec^5(x)dx}{\csc(x)}$

$$\int \frac{\sec^5(x)dx}{\csc(x)} = \int \frac{\frac{1}{\cos^5(x)dx}}{\frac{1}{\operatorname{sen}(x)}} = \int \frac{\operatorname{sen}(x)dx}{\cos^5(x)}$$

$Solución$: $u = \cos(x) \quad \rightarrow -du = \operatorname{sen}(x)dx$

$$\int \frac{\operatorname{sen}(x)dx}{\cos^5(x)} = \int \frac{-du}{u^5} + c = -\int u^{-5}du = (-)\frac{u^{-4}}{-4} + c$$

$$\frac{1}{4}u^{-4} + c = \frac{1}{4\,u^4} + c = \frac{1}{4\cos^4(x)} + c$$

$$\int \frac{\sec^5(x)dx}{\csc(x)} = \frac{1}{4}\sec^4(x) + c$$

42.$-Resolver$ $\int \frac{\cot(\ln(x))dx}{x}$

$Solución$: $u = \ln(x) \quad \rightarrow \quad du = \frac{dx}{x}$

$$\int \frac{\cot(\ln(x))dx}{x} = \int \cot(u)du = \ln(\operatorname{sen}(u)) + c = \ln\left(\operatorname{sen}(\ln(x))\right) + c$$

$$así\ que \quad \int \frac{\cot(\ln(x))dx}{x} = \ln\left(\operatorname{sen}(\ln(x))\right) + c$$

$ECUACIONES\ \ DIFERENCIALES$
$VARIABLES\ \ SEPARABLES$

$Una\ ecuación\ diferaencial\ de\ primer\ orden\ \dfrac{dy}{dx} = f(x,y)\ \ es\ separable\ si\ la\ función$

$f(x,y) se\ puede\ escribir\ como\ un\ producto\ de\ una\ función\ de$ "x" $y\ \ una\ función\ de$ "y", es

$decir,\quad \dfrac{dy}{dx} = f(x,y)\ \ \Rightarrow\ \ \dfrac{dy}{dx} = h(x)g(y).$

$Cuando\ esto\ ocurre, entonces\ es\ \ posile\ \dfrac{dy}{dx} = f(x,y)\ \ \Rightarrow\ \ \displaystyle\int \frac{dy}{g(y)} = \int h(x)dx$

$NOTA$: $Cada\ solución\ particular\ de\ una\ ecuación\ diferencial\ lineal\ de\ primer\ orden\ está$ $contenida\ en\ una\ solución\ general.\ \ Por\ contrario\ \ ,\ \ es\ común\ que\ una\ ecuación\ diferencial$ $no\ lineal\ de\ \ \ priner\ orden\ tenga\ una\ solución\ general\ que\ contenga\ una\ constante\ C\ \ como$ $una\ o\ varias\ soluciones\ \ particulares\ que\ no\ pueden\ obtenerse\ seleccionando\ un\ valor\ de\ C.$ $Estas\ soluciones\ \ excepcionales\ con\ frecuencia\ se\ denominan\ soluciones\ singulares\ o$ $soluciones\ perdidas.$

$Ejemplo$: $Resuelva\ la\ ecuación\ diferencial\ \ \dfrac{dy}{dx} = y^2.\cos(x)\ verificando\ si\ tiene\ solu-$ $ciones\ singulares\ o\ soluciones\ perdidas.$

$[no\ es\ lineal\ \ \ porque\ \ la\ mayor\ potencia\ de\ cada\ término\ en\ \ y\ \ \ no\ \ es\ 1\ (uno)]$

$$\frac{dy}{dx} = \ y^2.\cos(x)\ \ \Rightarrow\ \ \int \frac{dy}{y^2} = \int \cos{(x)}\,dx + c\quad \Rightarrow\ -\frac{1}{y} = \operatorname{sen}(x) + c$$

$$-\frac{1}{y} = \operatorname{sen}(x) + c\quad \Rightarrow\quad y = -\frac{1}{\operatorname{sen}(x) + c}(solución\ general).$$

$Verificar\ si\ \ \ y = 0\ \ \ \ es\ solución\ de\ la\ ecuación\ diferencial.$

$y = 0\ \ \ \rightarrow\ \ y' = 0\ \ ,\ \ \ al\ sustituir\ en\ la\ \ \ ED\ \ \ se\ \ obiene\ la\ identidad\ \ 0 = 0\ ,\ \ \ por\ la\ tanto$

$y = 0\ \ \ \ es\ \ solución\ de\ la\ ecuación\ diferencial.$

$Como\ no\ existe\ ningún\ valor\ de\ \ \ \ c \in R\ \ \ tal\ \ que\ \ y = 0\ \ entonces\ y = 0\ es\ una\ solución$

$perdida\ o\ singular\ de\ la\ ecuación\ diferencial\ \ \ \dfrac{dy}{dx} = y^2.\cos(x).$

$La\ solución\ será\ y(x) = -\dfrac{1}{\operatorname{sen}(x) + c} \quad y \qquad y = 0$

$1. -Resolver\ \dfrac{dy}{dx} = 6 - 3x^2$

$Solución\text{:}\ \dfrac{dy}{dx} = 6 - 3x^2 \ \Rightarrow\ dy = (6 - 3x^2)dx$

$$\int dy = \int 6dx - \int 3x^2dx + c \ \Rightarrow\ y = 6\int dx - 3\int x^2dx + c \ \Rightarrow\ y = 6x - x^3 + c$$

$2. -Resolver\ \dfrac{dy}{dx} = 3x^2y^2$

$Solución\text{:}\ \dfrac{dy}{dx} = 3x^2y^2 \ \Rightarrow\ \dfrac{dy}{y^2} = (3x^2)dx$

$$\int y^{-2}dy = \int (3x^2)dx + c \ \Rightarrow\ \frac{y^{-1}}{-1} = x^3 + c \ \Rightarrow\ -\frac{1}{y} = x^3 + c$$

$3. -Resolver\ \dfrac{dy}{dx} = x^{-2} - x$

$Solución\text{:}\ \ dy = (x^{-2} - x)dx \ \Rightarrow\ dy = x^{-2}dx - xdx$

$$\int dy = \int x^{-2}dx - \int xdx + c \Rightarrow\ y = \frac{x^{-1}}{-1} - \frac{x^2}{2} + c \quad \Rightarrow$$

$$y = -\frac{1}{x} - \frac{x^2}{2} + c \ \Rightarrow\ y = -\frac{2 + x^3}{2x} + c$$

$4. -Resolver\ \dfrac{dy}{dx} = \dfrac{x^2\sqrt{x^3 - 3}}{y^2}$

$Solución\text{:}\ \ y^2dy = \left(x^2\sqrt{x^3 - 3}\right)dx \ \Rightarrow\ \displaystyle\int y^2dy = \int x^2\sqrt{x^3 - 3}\ dx + c$

$$u = x^3 - 3 \quad \rightarrow \quad \frac{du}{3} = x^2dx$$

$$\int y^2dy = \frac{1}{3}\int u^{\frac{1}{2}}.du + c \ \Rightarrow\ \frac{y^3}{3} = \frac{2}{3}.\frac{u^{\frac{3}{2}}}{3} + c \ \Rightarrow\ y^3 = \frac{2}{3}\sqrt{(x^3 - 3)^3} + c_1$$

$5. -Resolver\ \dfrac{dy}{dt} = (2t + 1)^2$

$Solución\text{:}\ dy = (2t + 1)^2dt \ \Rightarrow\ \displaystyle\int dy = \int (2t + 1)^2dt + c$

$$Si \quad u = 2t + 1\,, \quad \frac{du}{2} = dt$$

$$\int dy = \int (2t+1)^2 dt + c \Rightarrow \int dy = \frac{1}{2}\int u^2 du + c \;\Rightarrow\; y = \frac{1}{6}u^3 + c \;\Rightarrow\; y = \frac{1}{6}(2t+1)^3 + c$$

$6.-Resolver \;\; \dfrac{dy}{dt} = 3t^3 + 4t - 6$

$Solución$: $\dfrac{dy}{dt} = 3t^3 + 4t - 6 \;\Rightarrow\; dy = (3t^3 + 4t - 6)dt$

$$\int dy = \int (3t^3 + 4t - 6)dt + c \;\Rightarrow\; y = 3\int t^3 dt + 4\int t dt - 6\int dt + c$$

$$y = \frac{3}{4}t^4 + 2t^2 - 6t + c$$

$7.-Resolver \;\; \dfrac{dy}{dx} = \dfrac{(x+2)^2}{(3-y)}$

$Solución$: $(3-y)dy = (x+2)^2\, dx \;\Rightarrow\; \int (3-y)dy = \int (x+2)^2\, dx \; + c$

$$u = 3 - y\;, \quad -du = dy\,, \quad v = (x+2)\,, \quad dv = dx$$

$$\int (3-y)dy = \int (x+2)^2\, dx + c \;\Rightarrow\; -\int u du = \int v^2 dv + c \;\Rightarrow\; -\frac{u^2}{2} = \frac{v^3}{3} + c$$

$$\frac{-3u^2 - 2v^3}{6} = c \quad\Rightarrow\quad 3(3-y)^2 + 2(x+2)^3 = c_1$$

$8.-Resolver \;\; y'' = \sqrt{2x-3}$

$Solución$: $\dfrac{d}{dx}(y') = (2x-3)^{\frac{1}{2}} \;\rightarrow\; d(y') = (2x-3)^{\frac{1}{2}}\, dx$

$$\int d(y') = \int (2x-3)^{\frac{1}{2}}\, dx + c_1$$

$$Si \quad u = (2x-3)\;, \quad \frac{du}{2} = dx$$

$$entonces \quad \int d(y') = \frac{1}{2}\int u^{\frac{1}{2}}\, du + c_1 \;\Rightarrow\; y' = \frac{2}{2.3}u^{\frac{3}{2}} + c_1$$

$$y' = \frac{1}{3}u^{\frac{3}{2}} + c_1 \;\Rightarrow\; \frac{dy}{dx} = \frac{1}{3}(2x-3)^{\frac{3}{2}} + c_1$$

$$dy = \left[\frac{1}{3}(2x-3)^{\frac{3}{2}} + c_1\right] dx \;\Rightarrow\; \int dy = \frac{1}{3}\int (2x-3)^{\frac{3}{2}}\, dx + c_1\int dx + c_2$$

$$\int dy = \frac{1}{3}\int (2x-3)^{\frac{3}{2}}\,dx + c_1\int dx$$

$$si \quad u = 2x-3 \ , \qquad \frac{du}{2} = dx$$

$$y = \frac{1}{6}\int u^{\frac{3}{2}}\,du + c_1\int dx + c_2 \ \rightarrow \ y = \frac{2}{6}\frac{u^{\frac{5}{2}}}{5} + \ c_1\,x + c_2$$

$$luego \quad y = \frac{1}{15}(2x-3)^{\frac{5}{2}} + c_1 x + c_2$$

$9.-Resolver\ \dfrac{d^2y}{dx^2} = 5x^2+1$

$Solución$: $\dfrac{d}{dx}(y') = (5x^2+1) \ \Rightarrow \ d(y') = (5x^2+1)dx$

$$\int d(y') = \int (5x^2+1)dx + \ c_1$$

$$y' = 5\int x^2dx + \int dx + c_1 \ \Rightarrow \ y' = \frac{5}{3}x^3 + x + c_1 \ \Rightarrow \ \frac{dy}{dx} = \frac{5}{3}x^3 + x + c_1$$

$$\int dy = \int \left(\frac{5}{3}x^3 + x + c_1\right)dx + c_2$$

$$y = \frac{5}{3}\int x^3dx + \int xdx + \int c_1dx + c_2 \ \Rightarrow \ y = \frac{5}{12}x^4 + \frac{x^2}{2} + c_1x + c_2$$

$10.-Resolver\ \ x^3\dfrac{dy}{dx} = y^2(x-4)$

$Solución$: $\quad y^{-2}dy = \dfrac{(x-4)}{x^3}dx \ \Rightarrow \ \displaystyle\int y^{-2}dy = \int\left(\frac{x-4}{x^3}\right)dx + c$

$$\int y^{-2}dy = \int \frac{x}{x^3}dx - \int \frac{4}{x^3}dx \ + c$$

$$-\frac{1}{y} = \int x^{-2}dx - 4\int x^{-3}dx + c \ \Rightarrow \ -\frac{1}{y} = \frac{x^{-1}}{-1} - 4\cdot\frac{x^{-2}}{-2} + c$$

$$-\frac{1}{y} = -\frac{1}{x} + \frac{2}{x^2} + c \ \Rightarrow \ -\frac{1}{y} = \frac{2}{x^2} - \frac{1}{x} + c \ \Rightarrow \ \frac{1}{y} = \frac{1}{x} - \frac{2}{x^2} + c$$

$$\frac{1}{y} = \frac{x-2}{x^2} + c$$

$11.-Resolver\ \dfrac{d^2y}{dx^2} = -9\,\mathrm{sen}(3x)$

$Solución$: $\dfrac{d(y')}{dx} = -9\,\mathrm{sen}(3x) \;\Rightarrow\; d(y') = -9\,\mathrm{sen}(3x)dx$

$$\int d(y') = -9\int \mathrm{sen}(3x)dx + c_1 \;\Rightarrow\; y' = (-9)\left(\frac{1}{3}\right)(-\cos(3x)) + c_1$$

$$y' = 3\cos(3x) + c_1 \;\Rightarrow\; \frac{dy}{dx} = 3\cos(3x) + c_1$$

$$\int dy = 3\int \cos(3x)dx + \int c_1 dx + c_2 \;\rightarrow\; y = \mathrm{sen}(3x) + c_1x + c_2$$

$12.-Resolver\ \ x^2y\dfrac{dy}{dx} = (y^2-1)^{\frac{3}{2}}$

$Solución$: $y(y^2-1)^{\frac{-3}{2}}dy = x^{-2}dx \;\Rightarrow\; \displaystyle\int y(y^2-1)^{-\frac{3}{2}}dy = \int x^{-2}dx\ + c$

$$-(y^2-1)^{-1/2} = \frac{x^{-1}}{-1} + c \;\Rightarrow\; -(y^2-1)^{-1/2} + \frac{1}{x} = c$$

$13.-Resolver\ \ \dfrac{dy}{dx} = (x+1)(x+2)\ \ ,para\ \ y = -\dfrac{3}{2}\ ,\qquad x = -3$

$Solución$: $\displaystyle\int dy = \int (x+1)(x+2)dx + c \;\Rightarrow\; \int dy = \int (x^2+3x+2)dx + c$

$$y = \frac{x^3}{3} + \frac{3x^2}{2} + 2x + c\ ,\quad como\ y = -\frac{3}{2}\ ,\ x = -3$$

$$Entonces\ -\frac{3}{2} = -9 + \frac{27}{2} - 6 + c \;\Rightarrow\; -\frac{3}{2} = -15 + \frac{27}{2} + c$$

$$-\frac{3}{2} = \frac{-30+27}{2} + c \;\Rightarrow\; -\frac{3}{2} = -\frac{3}{2} + c \;\Rightarrow\; c = 0$$

$$Luego\quad y = \frac{x^3}{3} + \frac{3x^2}{2} + 2x$$

$14.-Resolver\ \ (y+5)dx - (x-2)dy = 0$

$Solución$: $(y+5)dx - (x-2)dy = 0 \;\Rightarrow\; (x-2)dy - (y+5)dx = 0$

$$\frac{dy}{y+5} - \frac{dx}{x-2} = 0 \;\Rightarrow\; \int\frac{dy}{y+5} - \int\frac{dx}{x-2} = c \;\Rightarrow\; \ln(y+5) - \ln(x-2) = \ln C$$

$$\frac{y+5}{x-2} = C$$

$15.-Resolver\ \ xydx-(1+x^2)dy=0$

$Solución{:}\ \ xydx-(1+x^2)dy=0\ \ \Rightarrow\ (1+x^2)dy-xydx=0\ \Rightarrow\ \dfrac{dy}{y}-\dfrac{xdx}{1+x^2}=0$

$$\int\frac{dy}{y}-\frac{1}{2}\int\frac{2xdx}{1+x^2}=c\ \ \Rightarrow\ \ \ln y-\frac{1}{2}\ln(1+x^2)=c\ \ \Rightarrow\ \ \frac{y^2}{1+x^2}=C$$

$16.-Resolver\ \ (x-1)dy+\ y^2dx=0$

$Solución{:}\ \ (x-1)dy+\ y^2dx=0\ \ \Rightarrow\ \ \dfrac{dy}{y^2}+\dfrac{dx}{x-1}=0\ \ \Rightarrow\ \ \displaystyle\int\frac{dy}{y^2}+\int\frac{dx}{x-1}=c$

$$\frac{y^{-1}}{-1}+\ln(x-1)=-\ln C\ \ \Rightarrow\ -\frac{1}{y}=-\ln(x-1)-\ln C\ \ \Rightarrow\ \frac{1}{y}=\ln C(x-1)$$

$$y\ln C(x-1)=1$$

$17.-\ Resolver\ \ (x^2-2)dy+(2y+3)dx=0$

$Solución{:}\ \ (x^2-2)dy+(2y+3)dx=0\ \Rightarrow\ \dfrac{dy}{2y+3}+\dfrac{dx}{x^2-2}=0$

$$\frac{1}{2}\int\frac{2dy}{2y+3}+\int\frac{dx}{x^2-2}=\ln C\ \ \Rightarrow\ \ \frac{1}{2}\ln(2y+3)+\frac{1}{2\sqrt{2}}\ln\frac{x-\sqrt{2}}{x+\sqrt{2}}=C$$

$$\sqrt{2}\ln(2y+3)+\ln\frac{x-\sqrt{2}}{x+\sqrt{2}}=C$$

$18.-Resolver\ (x^2+4)xydy-(y^2+3)dx=0$

$Solución{:}\ \ (x^2+4)xydy-(y^2+3)dx=0\ \Rightarrow\ \dfrac{ydy}{y^2+3}-\dfrac{dx}{x(x^2+4)}=0$

$$\int\frac{ydy}{y^2+3}-\int\frac{dx}{x(x^2+4)}=\ln C\ \ \Rightarrow\ \ \frac{1}{2}\int\frac{2ydy}{y^2+3}-\int\frac{dx}{x(x^2+4)}=\ln C$$

$$\frac{1}{x(x^2+4)}=\frac{A}{x}+\frac{Bx+C}{x^2+4}\ \ \Rightarrow\ \ A=\frac{1}{4}\,,\qquad B=-\frac{1}{4}\,,\ \ C=0$$

$$\int \frac{dx}{x(x^2+4)} = \frac{1}{4}\int \frac{dx}{x} - \frac{1}{4}\int \frac{xdx}{x^2+4} = \frac{1}{4}\ln x - \frac{1}{8}\ln(x^2+4) + \ln C$$

$$Así\ \ que\ \int \frac{ydy}{y^2+3} - \int \frac{dx}{x(x^2+4)} = \frac{1}{2}\ln(y^2+3) - \frac{1}{4}\ln x + \frac{1}{8}\ln(x^2+4) = \ln C$$

$$4\ln(y^2+3) - 2\ln x + \ln(x^2+4) = \ln C \quad \Rightarrow \quad \frac{(y^2+3)^4(x^2+4)}{x^2} = C$$

$19. -Resolver\ \cos^2 y\, dx + \operatorname{sen} x\, dy = 0 \quad sabiendo\ que\ \ y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$

$Solución$: $\cos^2 y\, dx + \operatorname{sen} x\, dy = 0 \quad \Rightarrow \quad \frac{dx}{\operatorname{sen} x} + \frac{dy}{\cos^2 y} = 0$

$$\int \frac{dx}{\operatorname{sen} x} + \int \frac{dy}{\cos^2 y} = C \quad \Rightarrow \quad \int \csc x + \int \sec^2 y = C \quad \Rightarrow \quad \ln(\csc x - \cot x) + \tan y = C$$

$$como \quad \tan\left(\frac{\pi}{4}\right) = 1\,, \quad \cot\left(\frac{\pi}{4}\right) = 1\,, \quad \csc\left(\frac{\pi}{4}\right) = \sqrt{2}$$

$$\ln\left(\sqrt{2}-1\right) + 1 = C$$

$$entonces\ \ \ln(\csc x - \cot x) + \tan y = \ln\left(\sqrt{2}-1\right) + 1$$

$20. -Resolver\ la\ ecuación\ diferencial\ \ (3-y)dx + 2xydy = 0\ \ sabiendo\ que\ \ \ y(1) = 1$

$Solución$: $(3-y)dx + 2xydy = 0$

$$se\ divide\ por\ el\ factor\ \ 2x(3-y) \quad y \quad se\ \ tendrá$$

$$\frac{1}{2}\cdot\frac{dx}{x} + \frac{ydy}{3-y} = 0 \quad \Rightarrow \frac{1}{2}\cdot\frac{dx}{x} - \frac{-ydy}{3-y} = 0$$

$$\frac{1}{2}\int \frac{dx}{x} - \int \frac{-ydy}{3-y} = C \quad \Rightarrow \quad \frac{1}{2}\int \frac{dx}{x} - \left(1 + \frac{-3}{3-y}\right)dy = C$$

$$\frac{1}{2}\ln|x| - y - 3\ln|3-y| = C$$

$$como\ \ y(1) = 1 \quad entonces\ \ \frac{1}{2}\ln|1| - 1 - 3\ln|3-1| = C$$

$$C = -1 - 3\ln(2)$$

$$\frac{1}{2}\ln|x| - y - 3\ln|3-y| = -1 - 3\ln\,(2)$$